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Three-dimensional thermal stress analysis using the indirect BEM in conjunction with the radial integration method

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ABSTRACT

Thermal stress analysis is one of key aspects in mechanical design. Based on the indirect boundary integral equation (BIE) and the radial integration method (RIM), this paper develops a boundary-only element method for the boundary stress analysis of three-dimensional (3D) static thermoelastic problems. A transformation system constructed with the normal and two special tangential vectors is used to regularize the singularity in the indirect BIE. The RIM is then employed to transform the domain integrals arising in both displacement and its derivative integral equations into the equivalent boundary integrals, which results in a pure boundary discretized algorithm. Several numerical experiments are provided to verify the accuracy and convergence of the present approach.

1. Introduction

The boundary element method (BEM), as a powerful numerical technique, has been widely applied to the study of thermal stresses in three-dimensional (3D) thermoelastic problems [1–5]. The BEM transforms the differential equation to the boundary integral equation (BIE), which reduces the dimensionality of numerical problems by one. What's more, one can obtain more accurate numerical results by using the BEM compared with domain-type mesh methods, including the finite element method (FEM) and the finite difference method (FDM).

However, as we all know, the conventional BIEs have various orders of singular integrals arising from the application of the singular fundamental solution. How to accurately calculate singular integrals is one of most important issues for the numerical implementation of the BIEs [6–8]. The existing techniques for dealing with the singular integrals are separated into local and global approaches. The former approach directly converts singular integrals into non-singular form [9–16], whereas the later approach avoids singular integrals by establishing regularized BIEs [17–25]. The scope of this paper is focused on the global approach. Rudolphi [17] used two simple solutions to regularize the boundary integral equation of the normal derivative to a harmonic function. Liu and Rizzo [18] developed a weakly singular form of the hypersingular BIE for 3D elastic wave problems. Chen et al. [19,20] proposed the null-field integral equations in conjunction with degenerate kernels for multi-inclusion problem and stress field around circular holes under anti-plane shear. Zhang et al. [21,23] derived the regularized indirect BIE of displacement gradients for plane elastic and orthotropic elastic problems. After then, this indirect BEM strategy was extended to solve the thin structures for two dimensional thermoelastic problems [22]. Recently, Qu et al. [24] presented a non-singular indirect BEM formulation for three-dimensional potential gradient field. Compared with the direct BIEs, the indirect method is generally more accurate because its boundary element discretization only includes the source field. In addition, the indirect method does not contain the hypersingular integral, which indicates that its numerical evaluation is more easy and precise.

For thermoelastic problems, the direct application of the conventional BIEs generates a domain integral associated with the temperature of the material. To maintain the advantage of the boundary-only discretization for the BEM, numerous methods [5,26–33] have been developed to transform the domain integral into the equivalent boundary integral. The most popular one of these existing methods is the dual reciprocity method (DRM) [26–28]. The DRM employs a series of basis functions to approximate the boundary force effect quantities, and then converts the

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domain integrals into boundary ones by using particular solutions. The multiple reciprocity method (MRM) [30,31] is an extension of the DRM, which repeatedly utilizes a sequence of high-order Laplace operators to complete the transformation from the domain integral to the boundary integral. Recently, Gao [5,32,33] developed the radial integration method (RIM). This method is based on a pure mathematical treatment and therefore can transform any domain integral to the boundary integral in a uniform way.

In this paper, by combining the indirect BIE and the RIM, a boundary-only element method is proposed for the boundary stress analysis of 3D static thermoelastic problems. The present method employs a transformation system constructed with the normal and two special tangential vectors to remove the singularity in the indirect BIE, and then converts the domain integrals arising in both displacement and its derivative integral equations into the boundary integrals via the RIM technique. Finally, three benchmark examples are provided to verify the derived formulations. A brief outline of this paper is summarized as follows. Section 2 presents the problem definition and basic theorems. Section 3 derives the formulations of the regularized indirect BIE. Section 4 introduces the details of the RIM. Section 5 provides numerical examples to verify the present method. In Section 6, we conclude the paper.

2. Problem definition and basic theorems

2.1. Problem definition

The governing equation of three-dimensional (3D) static thermoelastic problems can be expressed in terms of displacements u_i as

$$(\lambda+G)u_{j,ij}(\mathbf{x})+Gu_{i,jj}(\mathbf{x})=(3\lambda+2G)\alpha T_{,i}(\mathbf{x}),\quad i,j=1,2,3,\ \mathbf{x}\in\Omega, \tag{1}$$

where T is the temperature, G the shear modulus, α the coefficient of thermal expansion, Ω the domain of interesting problems, and

$$\lambda=2G\frac{\nu}{1-2\nu}, \tag{2}$$

in which ν is the Poisson's ratio.

The Eq. (1) is solved by imposing the following boundary conditions

$$u_i=U_i(\mathbf{x}),\quad \mathbf{x}\in S, \tag{3}$$

or

$$\sigma_{ij}n_j=T_i(\mathbf{x}),\quad \mathbf{x}\in S, \tag{4}$$

where σ_{ij} is the boundary stress, $U_i(\mathbf{x})$ and $T_i(\mathbf{x})$ are known boundary function, and S is the boundary of the domain Ω.

The Kelvin fundamental solution [34,35] of the governing Eq. (1) of 3D static thermoelastic problems can be given as

$$u^*_{lk}(\mathbf{x},\mathbf{y})=\frac{1}{16\pi G(1-\nu)r}[(3-4\nu)\delta_{lk}+r_{,l}r_{,k}],\quad l,k=1,2,3, \tag{5}$$

where $\mathbf{x}=\mathbf{x}(x_1,x_2,x_3,)$ and $\mathbf{y}=\mathbf{y}(y_1,y_2,y_3)$ are the source and field points respectively, r denotes the Euclidean distance of $\mathbf{x}$ and $\mathbf{y}$, $r_{,l}=\partial r/\partial x_l (l=1,\ 2,\ 3)$ represent the derivatives of the distance r with respect to x_i, and

$$\delta_{lk}=\begin{cases}0, & l\neq k,\\ 1, & l=k.\end{cases} \tag{6}$$

For the traction, the fundamental solution is expressed as

$$p^*_{lk}(\mathbf{x},\mathbf{y})=-\frac{1}{8\pi(1-\nu)r^2}\times\left\{\frac{\partial r}{\partial\mathbf{n}}[(1-2\nu)\delta_{lk}+3r_{,l}r_{,k}]-(1-2\nu)[r_{,l}n_k-r_{,k}n_l]\right\},\quad l,k=1,2,3, \tag{7}$$

where $\mathbf{n}=(n_1,n_2,n_3)$ denotes a unit normal vector at point $\mathbf{x}$, and $\partial r/\partial\mathbf{n}=r_{,l}n_l$.

2.2. Basic theorems

In this section, we will provide some theorems used to derive the regularized BIEs. The first one is related with a transform formulation constructed based on the normal vector and two special tangential vectors, and the remaining ones are about the integral identities of fundamental solutions. We assume that $\mathbf{n}=(n_1,n_2,n_3)$ denotes a unit normal vector at a boundary point $\mathbf{x}$, $\mathbf{m}^1=(n_2+kn_3,-n_1,-kn_1)$ and $\mathbf{m}^2=(n_2,-n_1+n_3/k,-n_2/k)$ are two different vectors in the tangent plane of $\mathbf{x}$, and k is a constant ($k\neq 0$).

Theorem 1 [24]. *Let S be a piecewise smooth surface, $g(\mathbf{x})$ be a derivable function, and $(\mathbf{n},\mathbf{m}^1,\mathbf{m}^2)$ be a linearly independent set. Then we have*

$$\nabla g(\mathbf{x})=\mathbf{a}(\mathbf{x})\nabla g(\mathbf{x})\cdot\mathbf{m}^1+\mathbf{b}(\mathbf{x})\nabla g(\mathbf{x})\cdot\mathbf{m}^2+\mathbf{c}(\mathbf{x})\nabla g(\mathbf{x})\cdot\mathbf{n}, \tag{8}$$

in which $\nabla=(\partial/\partial x_1,\partial/\partial x_2,\partial/\partial x_3)$ and $a_i(\mathbf{x}), b_i(\mathbf{x}), c_i(\mathbf{x})\ (i=1,\ 2,\ 3)$ respectively are the components of vectors $\mathbf{a}(\mathbf{x})$, $\mathbf{b}(\mathbf{x})$, $\mathbf{c}(\mathbf{x})$. $a_i(\mathbf{x}), b_i(\mathbf{x}), c_i(\mathbf{x})\ (i=1,\ 2,\ 3)$ are given as

$$a_i(\mathbf{x})=\frac{(\boldsymbol{\delta}_i\times\mathbf{m}^2)\cdot\mathbf{n}}{(\mathbf{m}^1\times\mathbf{m}^2)\cdot\mathbf{n}},\ b_i(\mathbf{x})=\frac{(\mathbf{m}^1\times\boldsymbol{\delta}_i)\cdot\mathbf{n}}{(\mathbf{m}^1\times\mathbf{m}^2)\cdot\mathbf{n}},\ c_i(\mathbf{x})=\frac{(\mathbf{m}^1\times\mathbf{m}^2)\cdot\boldsymbol{\delta}_i}{(\mathbf{m}^1\times\mathbf{m}^2)\cdot\mathbf{n}}, \tag{9}$$

where $\boldsymbol{\delta}_i=(\delta_{i1},\delta_{i2},\delta_{i3}), i=1,\ 2,\ 3.$

Theorem 2 [24]. *Assume S is a piecewise smooth surface (open or closed), and $\hat{\mathbf{x}}$ is a point on S. Suppose $d=\{\inf|\mathbf{y}-\mathbf{x}||\mathbf{x}\in S\}$ and $h=|\mathbf{y}-\hat{\mathbf{x}}|$. If $\psi(\mathbf{x})\in C^{0,\alpha}(S)$ and $h/d\le K_1$ (K_1 is a constant), then there holds*

$$\lim_{\mathbf{y}\to\hat{\mathbf{x}}}\int_S\frac{x_k-y_k}{|\mathbf{x}-\mathbf{y}|^3}\left[\psi(\mathbf{x})-\psi(\hat{\mathbf{x}})\right]dS=\int_S\frac{x_k-\hat{x}_k}{|\mathbf{x}-\hat{\mathbf{x}}|^3}\left[\psi(\mathbf{x})-\psi(\hat{\mathbf{x}})\right]dS\ k=1,2,3. \tag{10}$$

Theorem 3. *Assume S is a piecewise smooth surface ($i=1$, 2and l, $k=1,\ 2,\ 3$), then we have*

$$\int_S\frac{\partial(r_{,l}r_{,k}/r)}{\partial\mathbf{n}(\mathbf{x})}dS=-4\pi\delta_{lk}\kappa(\mathbf{y})-2\int_S\frac{\partial(1/r)}{\partial x_k}n_l(\mathbf{x})dS,\quad \kappa(\mathbf{y})=\begin{cases}1, & \mathbf{y}\in\Omega,\\ 0, & \mathbf{y}\in\bar{\Omega},\end{cases} \tag{11}$$

$$\int_S\frac{\partial(1/r)}{\partial\mathbf{m}^i(\mathbf{x})}dS=0,\ \int_S\frac{\partial(r_{,l}r_{,k}/r)}{\partial\mathbf{m}^i(\mathbf{x})}dS=0,\quad \mathbf{y}\in\Omega\cup\bar{\Omega}, \tag{12}$$

where $\bar{\Omega}=R^3-(\Omega\cup S)$.

Proof. First, we set $B_\varepsilon(\mathbf{y})$ (or B_ε) is a sphere with the radius ε and its center at the point $\mathbf{y}$ in domain Ω. Then assume $\hat{\Omega}=\Omega-\Omega\cap\bar{B}_\varepsilon$, and $S_\varepsilon=\partial B_\varepsilon$.

As $l=k=1$, the left side of Eq. (11) can be given as

$$\int_S\frac{\partial(r_{,1}^2/r)}{\partial\mathbf{n}(\mathbf{x})}dS=-2\int_S\frac{\partial(1/r)}{\partial x_1}n_1(\mathbf{x})dS-3\int_S Pdx_2dx_3+Qdx_3dx_1+Rdx_1dx_2, \tag{13}$$