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Topological design optimization of lattice structures to maximize shear stiffness

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ABSTRACT

To improve the poor shear performance of periodic lattice structure consisting of hexagonal unit cells, this study develops a new computational design method to apply topology optimization to search the best topological layout for lattice structures with enhanced shear stiffness. The design optimization problem of micro-cellular material is formulated based on the properties of macrostructure to maximize the shear modulus under a prescribed volume constraint using the energy-based homogenization method. The aim is to determine the optimal distribution of material phase within the periodic unit cell of lattice structure. The proposed energy-based homogenization procedure utilizes the sensitivity filter technique, especially, a modified optimal algorithm is proposed to evolve the microstructure of lattice materials with distinct topological boundaries. A high shear stiffness structure is obtained by solving the optimization model. Then, the mechanical equivalent properties are obtained and compared with those of the hexagonal honeycomb sandwich structure using a theoretical approach and the finite element method (FEM) according to the optimized structure. It demonstrates the effectiveness of the proposed method in this paper. Finally, the structure is manufactured, and then the properties are tested. Results show that the shear stiffness and bearing properties of the optimized lattice structure is better than that of the traditional honeycomb sandwich structure. In general, the proposed method can be effectively applied to the design of periodic lattice structures with high shear resistance and super bearing property.

1. Introduction

The recent developments on large scale computation and advanced manufacturing techniques allow the construction of designs with fine and complex geometrical features [1], e.g. cellular and lattice structures, which have been increasingly investigated for a range of lightweight structural applications, including aerospace, automotive and naval industries due to their high stiffness-to-weight and strength-to-weight ratio, as well as their excellent energy absorption and thermal isolation characteristics [2–6]. As a representation, lattice materials are often assumed to be homogeneous due to it is a number of periodic units and can be efficiently assembled by following a certain pattern. Recently, the honeybee comb structure inspired by natural beeswax has long fascinated engineers and biologists for its outstanding mechanical properties and other characteristics, as a result of their special microstructures such as hexagonal, square and triangular, which are widely used in various engineering applications [7]. However, it has not proven until recently that this is the best structure with enhanced shear resistance that can be built with the beeswax to store the largest amount of honey [8].

Shear response is of great significance as bending lattice structure gives rise to transverse shear loading and may collapse the core of the structure in shear. For instance, the work [2] studied the effective in-plane stiffness of hexagonal honeycomb cores in macro-scale according to the bending model. After that, there is a growing interest in exploiting other materials as the filling material in such structures to simultaneously enhance the shear response, load-bearing and energy absorption capabilities of the lightweight structures, such as periodic lattice structure and honeycomb sandwich cores [9–11]. Many researchers [12–14] studied the mechanical properties of other honeycomb cores such as hollow pyramidal lattices, square honeycombs, and corrugated sandwich cores. Kelsey et al. [15] gave the upper and lower bounds of the shear modulus of honeycomb cores by using the unit displacement and unit load methods in conjunction with a simplification assumption for the strain and stress in the core. Pan et al. [16] and Han et al. [17] analyzed the longitudinal shear strength of honeycomb

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cores, and investigated the longitudinal shear deformation behavior and failure mechanism of aluminum alloy honeycomb cores at room temperatures. It is demonstrated that the shear or compression strength and specific energy absorption of lattice structures or sandwich cores can be increased dramatically by using appropriate filling foams. Also there have been some papers which studied the shear responses of various periodic lattice structures were mainly focused on the calculation and comparison of their shear modulus and experimental validations [18,19]. However, most of the lattice structures used in engineering are obtained by designers' experiences or inspired by existing materials or structures in nature. An effective method to guide the design of microstructural topologies of lattice materials is still in demand.

Inspired by the above natural-occurred cellular materials, the design of periodic lattice composite to achieve multifunctional properties (stiffness, strength, negative Poisson's ratio, etc.) based on the combination of structural optimization techniques and numerical homogenization has attracted considerable attention. For instance, Sigmund [20–22] employed a modified optimality criteria method to design material microstructure to achieve the prescribed properties. It can be found that the effective material properties at macro-scale are mainly determined by the topology of periodic microstructures, rather than the proportion and physical properties of their constituents, namely, the intrinsic material composition [23]. The homogenization method [24–25] has been recognized as a rigorous method for characterizing the macro mechanical behavior of materials consisting of periodic microstructures, under the assumption that the linearly elastic response of the periodic material can be determined by test strains over one cell [26]. The topological design of such materials assumes that the material is made of periodic cells which theoretically are infinitesimal within the structure, and the macro effective properties of the heterogeneous material are homogenized according to the microstructure that is the smallest repetitive unit of the material. The inverse design in the material is a typical topology optimization problem, which seeks an optimal microstructure of the cell with prescribed or extreme effective properties [27–36], and so on. A systematic mean of microstructural design is formulated as an optimization problem for parameters that represent the property and topology of the microstructure [37], so as to improve structural shear effect.

Over the past, topology optimization has been expanding as a powerful computational design tool for a range of structural and material problems both in academic research and industrial applications. Essentially, topology optimization is a numerical iterative process that distributes a given amount of material inside a reference design domain, so as to seek the best material layout until the expected performance is optimized subject to constraints [38]. Since the work of Sigmund [20–22], various structural topology optimization techniques have been developed for computationally design of microstructures of materials [33,39], e.g. the numerical homogenization method [40], solid isotropic material with penalization (SIMP) [41–43], level set method (LSM) [36], parametric level set method (PLSM) [44], and bidirectional evolutionary structural optimization (BESO) [45]. Amongst them, the design multifunctional composite materials are an active field of research [46]. Huang et al. [47] recently applied the BESO approach to the design of 2D and 3D materials with extreme bulk and shear modulus, and isotropic constraint is imposed by Radman [44]. Zhang et al. [48,49] and Xia et al. [50] proposed the strain energy-based method to predict the effective elastic properties, which compared with the numerical homogenization method and shows the advantages of higher computing efficiency and simplicity in the numerical implementation.

Based on the above work, we can find that there is still much room to improve the shear resistance of periodic lattice structure consisting of hexagonal unit cells in engineering applications. Lattice structures composed of unit cells are one of the candidates to enhance structural/material performances. However, the topological design problem of periodic lattice structure in recent works are mostly generalized by using the mathematical homogenization method, which has the disadvantages, such as the numerical implementation of the homogenization method and design sensitivity analysis are complicated and additional programming is needed. In this paper, the topology optimization technique is utilized to search the layout of periodic lattice structure with enhanced shear stiffness under the constraint of a given amount of material by using energy-based homogenization method. The optimization objective is to find the optimal distribution of two base materials within the unit cell of the periodic lattice material, so that the resulting structure has the maximum shear modulus. The effective properties of the microstructure are totally predicated by the periodic unit cell (PUC) which is discretized into finite elements under periodic boundary condition.

2. Energy-based homogenization method

2.1. Formulation and sensitivity analysis with energy-based homogenization

In general, the homogenization method applied to linear elastic problems establishes macroscopic properties that often are uniformly described by corresponding PUCs, which are periodically repeated in one or more directions. The properties of a heterogeneous medium rely on the analysis of its PUCs. Thus, the homogenization theory can be used to calculate the effective elastic constants of the macroscopic composite by the Hill-Mandel condition or energy averaging theorem [51]. To obtain the macroscopic equivalent constitutive properties by the homogenization process, three widely used types of loading can be applied: (a) prescribed linear displacements, (b) prescribed tractions and (c) periodic boundary conditions. The size of the periodic unit cell is assumed to be much smaller than the size of the bulk material, so that the average quantities of both the microscopic strain and stress can be defined [51].

Consider a single cell Y, the macro-scale displacement field μ is expanded, depending on the little aspect ratio ε between the macro and micro scales by using the asymptotic homogenization. When only the first-order terms of the asymptotic expansion are considered, the effective elasticity tensor of the macro-material can be found in terms of the material distribution in the domain of PUC by averaging the integral over the base cell [50] Y as:

$$C^H_{ijkl} = \frac{1}{|Y|}\int_Y \left(\varepsilon^{0(ij)}_{pq} - \varepsilon^{*(ij)}_{pq}\right) C_{pqrs}\left(\varepsilon^{0(kl)}_{pq} - \varepsilon^{*(kl)}_{pq}\right) \mathrm{d}Y \tag{1}$$

where $|Y|$ represents the area (or volume) of the unit cell, and $\varepsilon^{0(kl)}_{pq}$ defines the prescribed macroscopic strain fields, which are defined as three linearly independent unit strains: the horizontal unit strain $\varepsilon^{0(11)}_{pq} = [1 \quad 0 \quad 0]$, the vertical unit strain $\varepsilon^{0(22)}_{pq} = [0 \quad 1 \quad 0]$ and the shear unit strain $\varepsilon^{0(12)}_{pq} = [0 \quad 0 \quad 1]$. The local strain field $\varepsilon^{*(kl)}_{pq}$ induced by the test strains are defined as follow:

$$\varepsilon^{*(kl)}_{pq} = \varepsilon^{*}_{pq}\left(\chi^{kl}\right) = \frac{1}{2}\left(\chi^{kl}_{p,q} + \chi^{kl}_{q,p}\right) \tag{2}$$

where χ^{kl} denotes a Y-periodic admissible displacement field associated with the load case kl, which can be obtained from the following equilibrium equation [50]:

$$\int_Y C_{ijpq}\varepsilon^{*(kl)}_{pq}\frac{\partial v_i}{\partial y_j}\mathrm{d}Y = \int_Y C_{ijpq}\varepsilon^{0(kl)}_{pq}\frac{\partial v_i}{\partial y_j}\mathrm{d}Y \tag{3}$$

where v is the Y-periodic admissible displacement field.

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