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Patient-specific isogeometric structural analysis of aortic valve closure

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Abstract

We investigate the use of Isogeometric Analysis for the model construction and simulation of aortic valve closure. We obtain converged results and compare with benchmark finite element analysis. We find that Isogeometric Analysis is capable of attaining the same accuracy with models consisting of two orders of magnitude fewer nodes than finite element models; analogous savings are observed also in terms of analysis time. Model construction and mesh refinement are likewise performed more efficiently with Isogeometric Analysis.

Keywords: Isogeometric analysis; Aortic valve; Patient-specific modeling

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1. Introduction

The aortic valve regulates blood flow from the left ventricle to the aorta. Its crucial physiologic function and complex anatomy have motivated biomedical engineers to try to understand and explain aortic valve behavior using computer models. Even though the first aortic valve models date back to the 1970s [1–4], the complex physiological and pathological behavior of aortic valves have generated great contemporary interest [5–9]. In fact, the increase of population average age and of life expectancy makes heart valve disease and degeneration serious and constantly growing public-health problems, requiring appropriate resources to improve diagnosis and treatment [10].

The application of modern computational techniques for heart valve modeling represents a challenging research activity that may lead to the development of diagnostic tools, therapeutic devices, innovative prostheses, and the

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prediction of surgical outcomes. The purpose of the present work is to develop simple and accurate modeling and analysis procedures for the simulation of aortic valves. To accomplish this goal, we adopt isogeometric analysis (IgA) that may have significant advantages over other modeling and analysis procedures.

Introduced in 2005 [11], IgA employs typical functions from Computer Aided Design (such as, e.g., NURBS) for describing both geometry and field variables in an isoparametric fashion [12–14]. This leads to precise geometrical representations, typically exhibiting superior accuracy per degree of freedom with respect to finite elements [15–17, 12,18]. Since accurate results deriving from very precise geometrical descriptions and reduced computational times are key ingredients of computer-based simulations, especially in the field of predictive biomedicine, in this paper we aim at showing the suitability of isogeometric techniques for the simulation of aortic valves. In particular, starting from contrast-enhanced Computed Tomography Angiography (CTA) images in Dicom format, we obtain a stereolithographic (STL) representation of the aortic root, which is used as the target object to generate an isogeometric patient-specific model through a mapping procedure. Aortic leaflets, not visible with CTA, are constructed by integrating CTA data and ultrasound information, completing an aortic valve multi-patch structure. The diastolic behavior (corresponding to valve closure) of the aortic valve is calculated with isogeometric analysis. The “image-to-analysis” approach developed to perform patient-specific IgA of the aortic valve is sketched in the flowchart of Fig. 1 and compared to the corresponding one for finite element analysis (FEA).

The procedure to obtain a CAD representation of the aortic valve with the leaflets is the same in the two cases. However, following an IgA approach, the CAD representation is already an analysis-suitable model, while for finite elements a mesh generation step, that can be time consuming for complex geometries, is required. Additionally, when mesh refinement is necessary, with IgA a simple and inexpensive knot insertion (or degree elevation) operation at the CAD level is all that is required, while for finite elements a completely new mesh generation step has to be performed.

The paper is structured as follows: in Section 2, after presenting some basic concepts of IgA, we propose the approach to construct patient-specific geometries from medical images; in Section 3 we describe the isogeometric model of the aortic valve; and in Section 4 we present the simulation results. We draw conclusions in Section 5.

2. B-splines, NURBS and patient-specific modeling

Our procedure to obtain patient-specific surfaces is based on Non-Uniform Rational B-Splines (NURBS), which represent the dominant technology for design and geometric representation in industry. We also use NURBS for the approximation of field variables. In this section we give a very brief introduction to NURBS (details can be found in the book by Piegl and Tiller [19]) and to their use to construct an analysis-suitable geometrical model of the aortic valve starting from CTA images.

2.1. Basic concepts of B-splines and NURBS

NURBS are obtained from B-splines, which are piecewise polynomial curves composed of linear combinations of B-spline basis functions. A brief description of B-splines is therefore presented first.

B-spline basis functions are constructed from a set of parametric coordinates, collected in a *knot vector*, i.e., a non-decreasing sequence of real numbers: $\Xi = \{\xi_1, \ldots, \xi_{n+p+1}\}$, where p is the polynomial degree and n is the number of basis functions determined by the knots. The interval: $[\xi_1, \xi_{n+p+1}]$ is called a *patch*. A knot vector is called *open* if the first and last knot have multiplicity $p + 1$. In this case, the basis is interpolatory at the boundary points of the patch. The ith basis function of order p is defined recursively, starting from the piecewise constants ($p = 0$) according to the Cox–de-Boor recursion formula. For $p = 1, 2, 3, \ldots$ the basis functions are defined by:

$$N_i^p(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_i^{p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1}^{p-1}(\xi), \tag{1}$$

while for $p = 0$:

$$N_i^0(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1}, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Further details may be found in [12].