Advances in Engineering Software 107 (2017) 7–12

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Research paper

The application of drilling degree of freedom to checkerboards in structural topology optimization

B. Balogh, J. Lógó*

Budapest University of Technology and Economics, Department of Structural Mechanics, 1111 Budapest, Műegyetem rkp. 3, Hungary

ARTICLE INFO

Article history:
Received 28 September 2016
Revised 16 December 2016
Accepted 23 February 2017

Keywords:
Topology optimization
Optimality criteria
Checkerboard
Drilling
Allman
Micropolar

ABSTRACT

The subject of the paper is to investigate a new way for avoiding the development of checkerboard patterns in structural topology optimization, using an additional in-plane rotational freedom. The efficiency of a few, from the many existing formulations, with differing complexity are put into comparison, such as the standard 4 noded bilinear element, the Allman-type solution, the shell element from SAP2000 and finally an element constructed on the basis of micropolar theory. Since the emergence of checkerboarded regions is a general phenomenon, the optimization problem is as simple as possible, being a weight minimization with a compliance constraint, solved with the optimality criteria method and a FEM discretization of the design domain.

1. Introduction

The different types of numerical instabilities are well summarized in the article of Sigmund and Petersson [1], from which the focus is laid on the checkerboard pattern. The two examined case studies and the problem are illustrated in Figs. 1 and 2. It can be seen, that the domain has subdomains consisting of alternating solid and void elements. This pattern is generally due to mathematical instability [2,3], producing artificial stiffness, since this pattern does not correspond to an optimal distribution of material. Several papers investigated the problem and gave suggestions for the solution [4–8], such as smoothing, application of higher-order finite elements, patches, the Poulsen scheme, filtering, polygonal finite elements, the ground structure approach, etc. Our idea is that since classical continuum mechanics does not incorporate any intrinsic material length-scale parameter, it may be insufficient for the description of certain phenomena, such as a checkerboard-like material distribution above a given length-scale, because these microstructural regions bring non-classical behavior. *The fundamental assumption of this research is that the erroneous stiffness is due to the failure of the classical continuum-mechanical model, when applied to these types of structures.* As a remedy we suggest a novel approach, which is based on the application of drilling degrees of freedom, thus being strongly related to structural optimization problems. A natural way to incorporate such an additional DOF is the application of a higher order continuum theory, such as the Cosserat-theory. Besides that, we have investigated other ways, which are introduced through the sections of the paper, with increasing complexity. We expect, that the mere presence of in-plane rotations will eliminate the development of erroneous regions. Notedly, the existence of checkerboarded regions won't be a problem anymore, since the stiffness properties of that is reflected correctly by the theory. As far as possible, we aim to find finite elements with higher performance, where the application does not come with serious extra computational cost. Therefore we wish to find solutions where a general element remains four-noded and uses simple shape functions, as illustrated in Fig. 3.

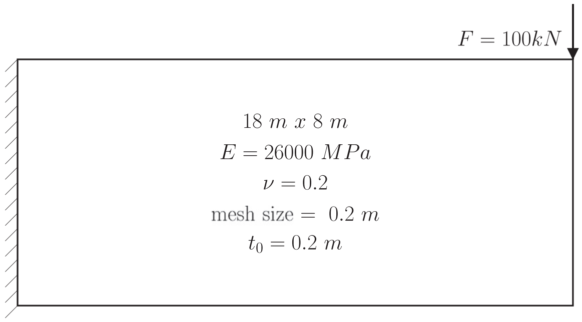
The mathematical problem to be solved can be stated in the following form:

$$\mathcal{P}_1 = \begin{cases} min_{t_1,\dots,t_n} & V(\mathbf{t}) = \sum_{i=1}^{n} A_i t_i & \text{(a)} \\ s.t. & \mathbf{u}^T\mathbf{K}(\mathbf{t})\mathbf{u} \le C_0 & \text{(b)} \\ & \mathbf{K}(\mathbf{t})\mathbf{u} = \mathbf{f} & \text{(c)} \\ & t_i \in \{0, t_0\}\ \forall i & \text{(d)} \end{cases} \qquad (1)$$

where $V(\mathbf{t})$ is the objective function representing the total volume, A_i denotes the area, t_i the thickness of the *ith* element of a finite element discretization, t_0 is the initial value for the thickness of the sheet, furthermore $\mathbf{K}(\mathbf{t})$, $\mathbf{u}$ and $\mathbf{f}$ are the stiffness matrix, the nodal displacement and force vectors of the structure, respectively. Since each finite element has the same area A_i, thus $A_i = A\ \forall i$, we simply refer to them as A in the following expressions. The term C_0 is an upper bound on the compliance of the linear system, which is necessary for the problem to be well posed. To avoid integer

* Corresponding author.
E-mail addresses: balogh.bence@epito.bme.hu (B. Balogh), logo.janos@epito.bme.hu, logo@ep-mech.me.bme.hu (J. Lógó).

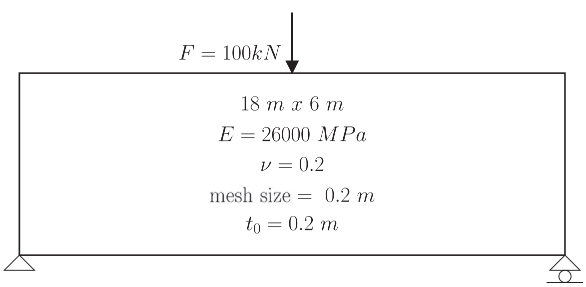
http://dx.doi.org/10.1016/j.advengsoft.2017.02.005



(a) Case problem 1

(b) Optimized structure 1

Fig. 1. The checkerboard phenomenon 1.



(a) Case problem 2

(b) Optimized structure 2

Fig. 2. The checkerboard phenomenon 2.

programming, we applied the popular power-low approach with a thickness penalization, where the intermediate thickness values are given by the initial thickness t_0 raised to some power p larger than one. As explained in detail following later in this section, the penalty parameter starts with the value 1.0, resembling the variable thickness shett problem, then it is gradually increased until reaching a maximum value. The continuous-variable problem has the following statement:

$$\mathcal{P}_2 = \begin{cases} min_{t_1,\ldots,t_n} & V(\mathbf{t}) \triangleq A\sum_{i=1}^{n} t_i^{1/p} & \text{(a)} \\ s.t. & c_1 \triangleq \mathbf{u}^T\mathbf{K}(\mathbf{t})\mathbf{u} - C_0 \leq 0 & \text{(b)} \\ & c_2 \triangleq \mathbf{K}(\mathbf{t})\mathbf{u} - \mathbf{f} = \mathbf{0} & \text{(c)} \\ & t_{min} \leq t_i \leq t_0 \ \forall i & \text{(d)} \end{cases} \quad (2)$$

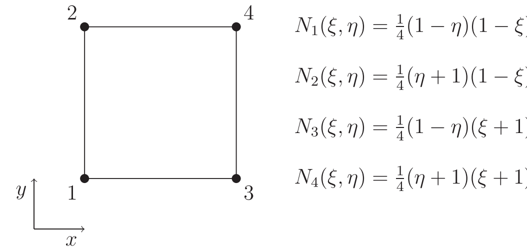


Fig. 3. Standard bilinear 4-node element.

Eqs. (2b) and (2c) are constraints on the design and the state variables, the latter one being the discrete state equation from a finite element solution of the governing equations of the 2D continuum. The constant $t_{\min}$ is a small number substituting zero, inserted to prevent singularity. The Lagrangian function for $\mathcal{P}_2$ is:

$$\Lambda(\mathbf{t},\lambda) = A\sum_{i=1}^{n} t_i^{1/p} - \lambda(\mathbf{u}^T\mathbf{K}(\mathbf{t})\mathbf{u} - C_0), \quad (3)$$

where λ is the Lagrange multiplier for the constraint. The necessary conditions for a local minimum (Karush–Kuhn–Tucker conditions) are:

$$\nabla_{\mathbf{t}}\Lambda(\mathbf{t},\lambda) = \mathbf{0} \quad (4a)$$

$$c_1 \leq 0 \quad (4b)$$

$$c_2 = 0 \quad (4c)$$

$$\lambda > 0 \quad (4d)$$

$$\lambda\ c_1 = 0 \quad (4e)$$

After constructing the Lagrangian function and applying Eqs. (4), the following formulas are obtained for the Lagrange-multiplier (λ) and the design variables (t_i):

$$\lambda = \frac{A\sum_{i=1}^{n} t_i^{1/p}}{C_0\ p}, \quad (5)$$

$$t_i = \frac{\lambda\ p\ C_i}{A} \qquad \forall i, \quad (6)$$

where C_i is the compliance of the ith element. The design variables, which satisfy the necessary conditions of a local minimizer are found by alternating Eqs. (5) and (6). The resulting iterational procedure can be classified as an alternating variables method and the steps are listed in Algorithm 1 (left arrow means assignment to the variable on the left-hand side).

In fact this heuristic iteration leads to a fix point type updating scheme, very similar to the one by Bendsøe, published in [9].

By penalizing not the density, but the thickness, intermediate values also have a physical interpretation. Furthermore, a complete justification of the power-law approach is given in [10]. According to (1d), the feasible region is bounded, and from the continuity of both the objective and the constraint functions the feasible region is closed. Furthermore, since problem $\mathcal{P}_1$ is posed in finite dimension, it has a solution in general. On the other hand, the statement of problem $\mathcal{P}_2$ encompasses a nonconvex objective function, when $p > 1$. In this case, the necessary conditions of a local minimizer only ensure that the resulting algorithm converges to a nearby stationary point, representing a better design.

1.1. The drilling degree of freedom

In the recent two decades there has been a great interest in elements possessing in-plane rotational degrees of freedom (also