Advances in Engineering Software 000 (2016) 1–12

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Eliminate localized eigenmodes in level set based topology optimization for the maximization of the first eigenfrequency of vibration

Zhenhua Li, Tielin Shi, Qi Xia*

The State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, China

ARTICLE INFO

Article history:
Received 17 August 2016
Revised 30 November 2016
Accepted 15 December 2016
Available online xxx

Keywords:
First eigenfrequency maximization
Topology optimization
Level set method
Body-fitted mesh
Mode recognition

ABSTRACT

Maximizing the fundamental eigenfrequency of vibration is an important topic in structural topology optimization. Previous studies of such a topology optimization problem should always be cautious of the "artificial localized mode" as it makes the optimization fail. In the present work, a level set based topology optimization is proposed to address such an issue. The finite element analysis is conducted on the actual structure by using a body-fitted mesh and without artificial weak material, thus localized mode that conventionally arises due to low-density region is prevented. In the present study, attention is turned to localized mode occurred gradually during the optimization. Such kind of localized mode results from the emergence of isolated area or cracked structure member produced by topological changes. A mode recognition technique based on the volume ratio of vibration-free region to the entire structure is proposed to identify such localized mode. Numerical examples of 2D structures are investigated.

1. Introduction

Structural topology optimization of the present study addresses the design of structure to improve its vibration performance. In the optimization, frequencies or frequency-based features are the essential concerns, and they are either treated as constraints [1–3] or as the objective function [4–6]. In each iteration of optimization, dynamic behavior of a design, for instance frequencies or mode shapes, is obtained through solving an eigenvalue problem, and then the design is improved based on the results of sensitivity analysis. In the end, the optimization results in a structure that has better dynamic performance.

Although much success has been achieved, a difficulty exists in the optimization of eigenfrequencies, i.e., the first eigenfrequency given by the FEA may belong to localized vibration of low-density regions mimicking void in the reference domain but not belong to the actual structure because the low-density regions are much more flexible than the actual structure [7]. This phenomenon is referred to as "artificial localized modes" . In such circumstances, optimization of the nominal first eigenfrequency of an artificial localized mode is meaningless. Similar difficulties will be also encountered when one deals with other eigenvalue optimization problems, for instance optimization of buckling-sensitive structure [8,9].

Many efforts have been made to address the issue mentioned above. One idea is to remove elements with minimum density from the reference domain, but re-appearance of material in the same locations becomes impossible [10]. Tenek and Hagiwara [5] set a minimum density threshold to avoid localized modes, but the optimization problem was changed from topology optimization to reinforcement optimization. Pedersen [7] investigated how the ratio between the penalization of mass and stiffness impacts the calculation of eigenvalue. Pederson proposed to: (1) linearize the ratio between the penalization of stiffness and mass in low density areas while maintaining the power law relation in other regions; (2) ignore the nodes in low density areas in the FEA. Tcherniak [11] set the mass of low density areas to zero, but this resulted in an overestimation of the corresponding eigenfrequency. Du and Olhoff [12] followed the above idea and proposed continuous interpolation model of mass matrix in low density area to avoid numerical singularity. Bogomolny [13] proposed to multiply the stiffness matrix by a polynomial, which consisted of penalized and non-penalized parts. Other interpolation model aiming at tailoring the ratio of the penalization to get localized-mode-free design can also be found in Bruggi and Venini [14], Huang et al. [15], and Rubio et al. [16]. In the above mentioned studies, through tuning the penalization of mass and stiffness in low density regions, the eigenfrequencies of localized modes were raised to be

* Corresponding author. Fax: +86-27-87559544.
E-mail address: qxia@mail.hust.edu.cn (Q. Xia).

http://dx.doi.org/10.1016/j.advengsoft.2016.12.001
0965-9978/

much higher than the first eigenfrequency of the actual structure. Therefore, these methods perfectly solved the aforementioned difficulties in the optimization of the first eigenfrequency.

The evolutionary structural optimization (ESO) method [17] and its modified version, bi-directional ESO (BESO) method [18], have also been used for the optimization of dynamic problems. In this method, the structure is modeled with finite elements, and several elements are removed or added after each eigenvalue analysis to optimize the frequency according to the contribution factor or sensitivity number [19–21]. Zhu et al. [22] extended the studies to use element replacement method to maximize the structural stiffness or natural frequency. As elements are removed from the design domain directly, artificial localized modes will not arise in hard-kill ESO/BESO method.

Recently, Guo et al. [23] proposed a computational framework for structural topology optimization based on the concept of moving morphable components (MMC). It finds the optimal structure topology by optimizing the shapes, length, thicknesses, orientations and layout of a set of morphable components directly [24], and it has been proved in solving problems with compliance and compliant mechanism design [25–27]. This method has the potential to avoid artificial localized modes and needs to be investigated in future work.

The level set method has also been used for the optimization of eigenfrequency. Osher and Santosa [28] studied the optimization of the first eigenvalue and spectral gap of a two-material drum. Allaire and Jouve [29] presented a solution to the maximization of the first eigenvalue in the form of Rayleigh quotient. Gournay [30] proposed a Hilbertian velocity extension method to improve the convergence rate and applied it to eigenvalue maximization. In our previous study, the level set method was used to maximize the simple and repeated first eigenvalue [31]. In the above studies, the artificial weak material was used to mimic void in the reference domain, and as aforementioned its material properties need to be carefully tailored, otherwise localized mode may appear.

Although the artificial weak material is used in conventional level set based topology optimizations, it is merely a convenient tool to simplify the FEA and is not indispensable. In this article, a level set based topology optimization problem is formulated in a "strict 0-1" manner by removing the artificial weak material and conducting the FEA with a body-fitted mesh, as proposed in our previous study [32]. Therefore, the localized modes due to the artificial weak material are essentially prevented.

However, localized modes still may arise during the course of optimization. For instance, an intermediate design containing isolated areas has several localized rigid-body modes whose eigenfrequencies are zero. There are two possible origins of such localized modes. First, drastic topological changes of a structure happening during the optimization may result in isolated area or cracked structure member. Second, similar situation will also be encountered when topological changes or local damage are intentionally created during the optimization, for instance when considering fail-safe or robust optimization aiming at a robust design whose performance is not disproportionately affected by local damage or uncertainty [33–35]. The localized modes of isolated area or cracked structure member with low eigenfrequency make the optimization discontinuous and meaningless. The method proposed in the present study addresses such localized modes by means of a mode recognition method.

The paper is organized as follows. Section 2.1 describes maximization of the first eigenfrequency of continuum structure and sensitivity analysis. Section 2.2 discusses localized modes arising in optimization process. Section 3 describes the FEA of structural vibration with body-fitted mesh and introduces mode recognition method to eliminate localized modes. Section 4 briefly introduces the level set method. Section 5 gives numerical examples and discussions. Section 6 concludes the paper.

2. Optimization problem

2.1. Problem description and sensitivity analysis

A structure is represented as an open bounded set $\Omega \subset R^d (d = 2 \text{or } 3)$, and the boundary of structure comprises two disjoint segments, i.e.

$$\partial\Omega = \Gamma_N \cup \Gamma_D$$

where a Dirichlet boundary condition is imposed on Γ_D, a Neumann boundary condition on Γ_N. During optimization all admissible designs should stay in a fixed reference domain $D \subset R^d, \Omega \subset D$. The weak form of the eigenvalue problem of linear elastic structure vibration is written as

$$a(u, \upsilon) = \lambda b(u, \upsilon), \forall \upsilon \in U \tag{1}$$

where λ is the eigenvalue; u is the eigenvector; $U = \{\upsilon \in H^1(\Omega)^d \;|\upsilon = 0 \text{on} \Gamma_D\}$ is the space of kinematically admissible displacement fields; $a(u, \upsilon)$ and $b(u, \upsilon)$ are defined as

$$a(u, \upsilon) = \int_\Omega Ae(u) \cdot e(\upsilon) d\Omega \tag{2}$$

$$b(u, \upsilon) = \int_\Omega \rho u \upsilon \; d\Omega \tag{3}$$

where A is the stiffness tensor; $e(u)$ is the strain tensor; ρ is the material density. Generally, the eigenvectors are mass-orthonormalized by imposing the condition

$$b\left(u_i, u_j\right) = \delta_{ij} \tag{4}$$

where δ_{ij} is the Kronecker Delta.

The optimization problem of present work is to maximize the first eigenfrequency λ_1 subject to a constraint of volume, i.e.,

$$\begin{aligned} &\max \lambda_1 \\ &\text{s.t. } a(u, \upsilon) = \lambda_1 b(u, \upsilon), \forall \upsilon \in U \\ &V - \bar{V} \le 0 \end{aligned} \tag{5}$$

where $V = \int_\Omega d\Omega$ is the volume of structure, and $\bar{V}$ is the upper bound of volume.

For the completeness of the present paper, the sensitivity analysis for the optimization problem [31] is briefly revisited in the remaining part of this section. The Lagrangian of the optimization problem is defined as

$$L = \lambda_1 + a(u, w) - \lambda_1 b(u, w) + \ell\left(\bar{V} - V\right) \tag{6}$$

where $w \in U$ is a Lagrange multiplier for the equation of eigenvalue problem Eq. (1); ℓ is the Lagrange multiplier for the volume constraint, and it is updated during optimization according to the Augmented Lagrange multiplier method [36] as

$$\ell^{k+1} = \max\left\{0, \ell^k + \frac{1}{\mu}\left(\int_\Omega dx - \bar{V}\right)\right\} \tag{7}$$

where μ is a penalization factor close to zero and set by designer.

The material derivative of the Lagrangian is given by

$$L' = \lambda_1{}' + a'(u, w) - \lambda_1{}' b(u, w) - \lambda_1 b'(u, w) - \ell V' \tag{8}$$

where $a'(u, w)$, $b'(u, w)$ and V' are given as

$$a'(u, w) = a\left(u', w\right) + a\left(u, w'\right) + \int_{\partial\Omega} Ae(u) \cdot e(w) V_n d\Gamma \tag{9}$$

$$b'(u, w) = b\left(u', w\right) + b\left(u, w'\right) + \int_{\partial\Omega} \rho u w V_n d\Gamma \tag{10}$$

$$V' = \int_{\partial\Omega} V_n d\Gamma \tag{11}$$