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Two-dimensional domain decomposition based on skeleton computation for parameterization and isogeometric analysis

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Highlights

- Two-dimensional domains are decomposed into subdomains based on skeleton computation.
- Domain partition provides better parameterization than a one-patch representation.
- The new parameterization is superior to other techniques in isogeometric analysis.

Abstract

This paper proposes a method for decomposing two-dimensional domains into subdomains for parameterization and isogeometric analysis. Given a complex domain in a plane, the skeleton of the domain is computed to guide the domain decomposition. A continuous parameterization of the domain is then obtained by parameterizing each respective subdomain. This parameterization method is applied with isogeometric analysis to solve numerical PDEs over two-dimensional domains. Examples are provided to demonstrate that the new parameterization method is superior to other state-of-the-art parameterization techniques and that it performs better in isogeometric analysis.

Keywords: Parameterization; Isogeometric analysis; Skeleton; Domain decomposition

1. Introduction

Isogeometric analysis (IGA), a method recently proposed by T.J.R. Hughes et al. [1], is a new framework for use with the finite element method (FEM) that integrates two related disciplines: computer-aided-engineering (CAE) and computer-aided-design (CAD). In IGA, geometries are precisely represented by parametric equations that remain unchanged throughout the process of refinement so that problems that are sensitive to geometric imperfections can be more readily solved [2]. IGA overcomes many problems encountered with FEM, such as mesh generation and mesh refinement. IGA has been successfully implemented in many areas, such as linear elasticity, shell problems, structural vibrations, electromagnetics, optimization and phase transition phenomena [3–6,2,7–9].

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IGA uses consistent basis functions for the geometrical representation and numerical computation of PDEs. A common geometrical representation in CAD is the non-uniform rational B-splines (NURBS). However, NURBS do not have local refinement property. To facilitate adaptive solutions to PDEs, various local refinement splines have been developed, such as hierarchical splines, T-splines, PHT splines and LR splines [10–15]. Methods for constructing and analyzing suitable local refinement splines are subject to active research.

Given a computational domain over which a PDE is solved, a central problem in IGA is to compute a good parametric representation for the domain. This problem is called *parameterization*. The parameterization greatly influences the numerical accuracy and efficiency of the numerical solutions. Several approaches have been proposed to solve the parameterization problem. A common method is to use harmonic mapping to map a square to a computational domain using B-splines [16–18]. A recent study [18] proposed a method to create trivariate representations with B-splines or T-splines as an extension of the method proposed in [17]. Aigner, et al. [19] presented a variational framework for generating NURBS parameterizations of swept volumes, in which the control points can be obtained by solving an optimization problem. Another proposed method uses parameterization of a 2D domain with four planar boundary B-spline curves by solving a constraint optimization problem [20]. It has also been demonstrated that the quality of different parameterization methods can influence the solutions of the PDEs in IGA [21].

All of the above methods were proposed for finding a global parameterization of a computational domain. However, for complex domains, it is very hard or even impossible to find a single global parameterization using splines. Moreover, even when parameterization is possible, the quality of the parameterization can be very low, which is not desirable for isogeometric analysis. In this paper, we propose a totally new idea to solve this problem. First, a computational domain is decomposed into subdomains using the skeleton of the domain as a guide. Each subdomain is then parameterized to obtain a continuous parameterization for the computational domain.

This paper is organized as follows. In Section 2, we present an example to show that domain decomposition can improve the efficiency and accuracy of PDE solutions in IGA. In Section 3, a method is proposed to decompose a computational domain into subdomains. In Section 4, we present an algorithm for the parameterization of a computational domain based on the parameterization of each subdomain. Some examples of IGA based on our parameterization technique are given in Section 5. Section 6 concludes the paper with some future work.

2. Stationary heat conduction: L-shaped domain

The problem of stationary heat conduction in the L-shaped domain has been examined previously [22,23]. We will use this problem to show that domain partitioning can sometimes greatly improve the numerical accuracy of PDE solutions when using IGA.

The L-shaped domain is $\Omega = [-1, 1] \times [-1, 1] \setminus [0, 1] \times [0, 1]$ and the control equation is

$$-\Delta u = 0 \quad \text{in } \Omega,$$

with a homogeneous boundary condition for $\Gamma|_D$ and a Neumann boundary condition

$$\frac{\partial u}{\partial \mathbf{n}} = \frac{\partial f}{\partial \mathbf{n}} \quad \text{on } \Gamma_N,$$

as shown in Fig. 1, where $\mathbf{n}$ is the outer normal vector. The domain is concave, and the solution, which is singular at the origin, belongs to the space $\mathbf{H}^s = \{f \in L^2(\Omega) \mid \mathbb{D}^\alpha f \in L^2(\Omega), |\alpha| \leqslant s\}$ for $0 < s < \frac{2}{3}$. We solve the problem using IGA. We will show that different parameterizations of the L-shaped domain Ω result in different convergence properties of the solution.

The L-shaped domain Ω can be parameterized by a single biquadratic B-spline, as shown in Fig. 3. We can also partition Ω into two subdomains, as shown in Fig. 2, and parameterize each subdomain with a biquadratic B-spline to obtain a two-patch parameterization of Ω (Fig. 4). Figs. 5 and 6 show the solution and L^2 error for our parameterization. The convergence results based on these two parameterizations behave quite differently. Table 1 summarizes the convergence results based on the two parameterizations and Fig. 7 depicts the convergence plots. The numerical solution based on the parameterization after the domain decomposition converges faster than the parameterization without decomposition. The L^2 error of the solution using the former parameterization is about one tenth of the L^2 error of the latter parameterization with about the same number of degrees of freedom. This example sufficiently demonstrates that domain decomposition can improve the parameterization and numerical accuracy of