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A sharp interface isogeometric solution to the Stefan problem

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Highlights

- Enriched Isogeometric analysis for the Stefan problem.
- Sharp interface solution to both classical and dendritic Stefan problems.
- Algebraic distance estimations and point projection algorithms developed for efficient analysis.
- Direct imposition of Stefan and Gibbs–Thomson conditions.
- Demonstrated with several numerical examples of classical and dendritic Stefan problems.

Abstract

In the present paper, the Stefan problem is solved by enriching an underlying NURBS-based isogeometric approximation with an explicitly defined (sharp) interface on which a hybrid function/derivative condition is isoparametrically described. Since the geometry of the enrichment is explicitly defined, normals and curvatures are explicitly computed at any point on the interface. Thus, the enriched approximation naturally captures the interfacial discontinuity in temperature gradient and naturally enables the imposition of Gibbs–Thomson condition. The blending of the enrichment with the underlying approximation requires an estimate of distance to the enriching geometry from a quadrature point and the parametric value of the footpoint on the enriching geometry. These quantities are computed efficiently in the present paper using an algebraic estimate of distance coupled with an algebraic point projection method. These algebraic schemes rely on implicitization of the parametric curve, and are shown to be more efficient and robust than Newton–Raphson iterations. Procedures for adaptive time stepping, refinement and coarsening of geometry are developed to increase the stability and efficiency of the developed methodology. Several numerical examples of classical and dendritic Stefan problem are presented to demonstrate the methodology.

Keywords: Stefan problem; Hybrid function/derivative enrichments; Algebraic distance and point projection; Adaptive geometry and time stepping; Dendritic solidification

1. Introduction

The Stefan problem, mathematically describing solidification or melting [1], is a moving boundary problem of importance in many engineering applications. A significant difficulty in solving the Stefan problem, in addition to computationally modeling the moving interface, is in applying the two interface conditions: the Stefan condition and

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Nomenclature

Ω	Problem domain
Ω_l, Ω_s	Liquid and solid sub-domains
Γ_{int}	Interface between Ω_l and Ω_s
Γ_T, Γ_q	Dirichlet and Neumann boundary
$\bar{T}$	Known temperature on Dirichlet boundary
$\bar{q}$	Known normal heat flux on Neumann boundary
$\mathbf{n}$	Normal to the interface
$\mathbf{n}_l$, $\mathbf{n}_s$	Outward normal to Ω_l and Ω_s
t	Time
T	Temperature field
T_l, T_s	Temperature field in Ω_l and Ω_s
ρ	Material density
c	Specific heat
c_l, c_s	Specific heat for Ω_l and Ω_s
k	Thermal conductivity
k_s, k_l	Thermal conductivity in Ω_l and Ω_s
s	Heat source
$\mathbf{x}$	Point in the physical space
L	Latent heat (per unit mass)
q_n	Normal heat flux
q_{n_l}, q_{n_s}	Normal heat flux into Ω_l and Ω_s
v_n	Normal interfacial velocity
T_m	Melting temperature
κ	Mean surface curvature
γ	Capillary length
ϵ_c	Surface tension coefficient
ϵ_v	Kinetic mobility coefficient
u	Parameter of geometric entity
u_f	Parameter value of footpoint
$\mathbf{C}(u)$	Parametric curve or surface entity (u is a vector for a surface)
$\mathcal{P}(\mathbf{x})$	Function projecting $\mathbf{x}$ onto lower dimensional parametric entity $\mathbf{C}(u)$
$d(\mathbf{x})$	Distance between $\mathbf{x}$ and lower dimensional parametric entity $\mathbf{C}(u)$
w	Weight function
d_s	Scaling factor for weight function
$f(\mathbf{x})$	Behavioral field
$f_\Omega(\mathbf{x})$	Approximation to behavior over domain
$f_\Gamma(u)$	Approximation to behavior on enrichment
T^c	Approximation to temperature field on underlying domain
T^e	Enriching temperature field defined isogeometrically on interface
G_i^e	Enriching temperature gradient defined isogeometrically on interface
G_l^e, G_s^e	Enriching temperature gradient corresponding to liquid or solid subdomains
$P_I(x_I, y_I, z_I)$	Ith Control point coordinates
n, n_e	Number of degrees of freedom for domain approximation and enrichment
N_I^c, N_I^e	NURBS basis function for domain approximation and enrichment corresponding to control point P_I
$\bar{v}_I$	Speed value associated with control point P_I
$\mathbf{v}_I(v_I^x, v_I^y, v_I^z)$	Velocity of control point P_I
T_i	Initial temperature
T_w	Wall temperature