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Research paper

An analytic meshless enrichment function for handling discontinuities in interactive surgical simulation

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ABSTRACT

For surgical simulation applications, realistic behavioral modeling of soft tissue is considered to be one of the most significant challenges, because biomechanical soft-tissue models need to reflect the correct elastic response, be efficient in order to run at interactive simulation rates, and be able to support operations such as cuts and sutures. For these reasons, having a usable 3D cutting engine is a significant feature for interactive surgery simulation software. Mesh-based solutions, where the connections between the individual degrees of freedom (DoF) are defined explicitly, have been the traditional approach to soft-tissue biomechanics. However, when the problem under investigation in interactive biomechanics contains a simulated surgical gesture that entails a cut that disrupts the connectivity, the underlying mesh structure has to undergo remeshing operation, and most of the time it causes the performance bottleneck in the simulation. Unlike the tightly-coupled nonoverlapping element composition of the mesh-based solutions, this paper builds an analytic enrichment function on top of a loosely-coupled meshless method for constitutive modeling of elastic soft tissues, where arbitrary discontinuities or cuts are applied to the objects in the context of surgical simulation. Enrichment values for a continuous cut interface are computed and stored inside a grid structure that is accessed by individual meshless nodes in order to update their weight functions. The presented analytic enrichment function is efficient to compute and easy to integrate into existing meshless models. The meshless mechanics code and the enrichment-based cut handling functionalities have been implemented within the open-source simulation framework SOFA.

1. Introduction

Medical education traditionally involves the concept of apprenticeship [1], where novices directly learn from experienced doctors, while gradually taking an increasing role in therapy provided to patients to increase their level of expertise. Although classical apprenticeship program constitutes the basis of the medical education field, relying solely on it is not an optimal education strategy as it requires long training hours to bring the expertise to the desired level and also it is difficult to ensure that trainees experience all types of major cases. Computer-based modeling and simulation practices such as virtual reality surgery simulation have begun to make an impact in order to alleviate the aforementioned shortcomings of the traditional medical education.

Computer-based medical modeling and simulation features both anatomical models and therapy models, both of which represent significant challenges. One of the greatest challenges in building complex therapy models is to capture the accurate response of soft tissue [2–4]. For surgical simulator applications, biomechanical models of human soft tissues have to be accurate, efficient enough to be computed in real time, and able to handle topology-altering operations such as cuts and sutures [5]. Volumetric cutting of soft tissue is essential to interactive surgery simulation and convincing implementation of volumetric cutting is still an active research area in surgical simulation.

This paper presents a new approach for treating material discontinuities such as cuts, which is built on top of a point-based rather than mesh-based method. The introduced cuts are handled through mathematical structures called *analytic enrichment functions*, which are essentially functions that are discontinuous across the cut edge yet smoothly varying around the tip of the cut. Enrichment functions also need to be efficient enough to be computed on-the-fly while a cut is continuously being introduced in the simulation domain. The cut that affects the deformable body is represented as a piecewise linear segment, which defines a local cut-centered coordinate system that is used to compute the associated enrichment function. The enrichment values for a continuous cut series are computed and stored in a grid structure that is called the *Enrichment Grid*. The Enrichment Grid also doubles as a

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useful data structure in order to accelerate intra-simulation steps such as intersection queries. The methods and the algorithms described in this paper are implemented as an extension to the popular open-source medical simulation framework Simulation Open Framework Architecture (SOFA) [6]. Our contribution to the SOFA codebase, which was previously lacking a volumetric cutting algorithm, is a significant contribution to the open-source surgery simulation community.

The rest of the paper is organized as follows. Section 2 provides a brief overview of meshless methods in general. Section 3 describes various strategies in handling discontinuities in meshless methods. Section 4 presents our contribution, which improves on previous meshless approaches, and introduces the Enrichment Grid data structure. Section 5 provides the point-based deformable object modeling along with the extended enrichment grid approach. Finally, Section 6 concludes the paper by presenting the final remarks.

2. Previous work on tissue deformation and cutting

Deformable modeling of soft tissue is a continuum elasticity problem, whose numeric solution involves the discretization of a continuous domain into discrete elements. Numerous non-physical and physically-based models have been utilized in order to approximate this solution, which typically rely on an underlying mesh structures either in 2D or 3D depending on the nature and the requirements of the problem. A breadth-first classification of mesh-based continuum models includes mass-spring networks [7], finite element methods [8], finite volume methods [9], and finite difference methods [10]. Among these, the finite element method has received particular interest in the biomechanical modeling community.

The early work of Bro-Nielsen discussed a fast adaptation of finite element modeling to satisfy speed and robustness requirements in a surgical simulation setting [11]. In this framework, the author incorporated a technique called condensation, which translates into obtaining a more compact version of the system model by rearranging or eliminating terms of the matrix equations by simplifying a volume into a system of boundary elements. The accuracy of the condensation procedure largely depends on the redistribution quality of the masses; in case of a non-optimal distribution, the solution accuracy can be adversely affected [8]. Moreover, this type of simplification is incompatible with arbitrary cutting.

Another technique developed to optimize the fidelity versus efficiency trade-off is the finite element model based on Total Lagrangian Explicit Dynamics (TLED) by Miller et al. [12]. The difference between the TLED based finite element model and other approaches is the former's use of the original reference configuration of the object to calculate the stress and strain tensors during a simulation step. As a result of expressing computations in the reference coordinates, the authors were able to pre-compute spatial derivatives. The pre-computation of the spatial derivatives leads to efficiency in terms of computational resources, while being capable of handling geometric and material non-linearities. The authors employed central differences-based explicit integration rather than the implicit integration scheme. With this choice, they were able to avoid solving the set of non-linear algebraic equations that are required by the implicit integration at each time step. However, the use of explicit integration entails limitations on the time step size in order to ensure the stability of the system. The authors justified their implementation choice by stating that the relatively lower stiffness (Youngs modulus) value of the soft tissue relaxes the time step limitation considerably compared to the typical simulations involving stiffer material like steel or concrete.

Another attempt to increase the computational efficiency of the elastic model in the context of interactive simulation was discussed in the method proposed by Marchesseau et al. [13]. The authors presented a new discretization method called Multiplicative Jacobian Energy Decomposition (MJED), which allows the simulation to assemble the stiffness matrix of the system faster than the traditional Galerkin FEM formulation. The method utilized an implicit solver with larger time steps, which has the potential of producing more stable simulations, in application areas that involve haptic interactions. The authors reported computation accelerations of up to five times for the St. Venant Kirchoff materials. TLED and MJED methods both rely on pre-computation of simulation variables in order to achieve faster solutions at each time step. Although useful for modeling the elastic response of the deformable body that does not involve topological changes, these pre-computations at the initial configuration of the simulation object would be invalidated when a topology-changing cut is introduced to the system. In other words, TLED and MJED are undermined by interactive cutting requirements.

Free-form deformation lattices, mass-spring networks, and finite element models (FEM) that are composed of tetrahedral/hexahedral elements are all examples of mesh-based models that result in systems with many degrees of freedom (DoFs) that essentially define the total kinematic state of the modeled object. The aforementioned model examples have one property in common, they all define the connectivity information between the DoFs explicitly. When there is a situation that disrupts this connectivity, such as an introduced discontinuity in the form of a cut, the discretization of the continuum needs to be redefined to handle the changes in the connectivity. Various approaches have been proposed to handle these changes caused by cuts. Courtecuisse et al. [14] presented an FEM-based soft tissue deformation methodology that also supports real-time virtual cutting. In the presence of a cut, the topology of the finite elements comprising the simulation object changes along with the simulation-specific matrices. The topology changes in Courtecuisse's implementation were encoded in three types of topology operations: element removal, element subdivision, and element addition. This work benefitted from a GPU-based parallel implementation in order to ensure interactive operation rates.

More recently, Wu et al. [15] discretized the simulation object by using a semi-regular hexahedral finite element grid. The volume was partitioned using an octree, and the face-adjacent cells of the octree were linked together. The advantage of this discretization is the ability to update the topology of the elements in an efficient way, when a cut is being introduced to the simulation domain, by marking the links between the affected elements as disconnected. The octree was refined dynamically along the cut surface in order to retain fine detailed cuts. The authors employed several approximations of the deformable model, such as the concept of Composite Finite Elements (CFEs), in which smaller neighboring hexahedral elements are grouped together to form larger elements, thus decreasing the number of DoFs significantly. With this CFE-based approximation and a multigrid implicit solver [15], the authors were able to achieve simulation rates of 15 frames per second during the cutting operation.

3. Meshless methods overview

Mesh-based methods such as FEM have been widely used for modeling physical phenomena such as elasticity, heat transfer, and electromagnetism, while relying on the assumption of a continuous domain. However, FEM is not well suited to problems involving extreme mesh distortions that result in degenerate element shapes, moving discontinuities that do not align with the element edges such as propagating cracks or cuts [17], and advanced