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Adaptive hexahedral mesh generation and regeneration using an improved grid-based method

Lili Huang[a,*], Guoqun Zhao[b], Zhonglei Wang[b], Xiangwei Zhang[c]

[a] Institute of Engineering Mechanics, Shandong Jianzhu University, Jinan, Shandong 250101, P.R. China
[b] Engineering Research Center for Mould & Die Technologies, Shandong University, Jinan, Shandong 250061, P.R. China
[c] School of Management, Shandong University, Jinan Shandong 250100, P.R. China

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ABSTRACT

An improved grid-based algorithm for the adaptive generation and regeneration of hexahedral element mesh is presented in this paper. The method for the mesh density generation and control is introduced. The refinement field is generated based on the surface curvatures, geometry features, density windows and field variables distribution. To give good description of the geometry features, eight different types of free element facet configurations are given for the mesh matching to the surface of the solid model. *Scaled Jacobian* and the *Condition Number* of the Jacobian matrix are used to evaluate the hexahedral element mesh quality. A curvature-based Laplacian smoothing approach is employed to improve the quality of boundary elements and preserve the boundary characters of mesh. To improve the quality of the surface meshes and the interior elements, an optimization approach is proposed by using mesh quality metric as the objective function. By combining the Laplacian smoothing method with the optimization approach, the mesh quality is improved significantly. For the surface meshes, the *Condition Number* of a set of Jacobian metric associated with the quadrilateral elements is taken as the optimization objective function. For the interior elements, the *Condition Number* metric associated with the hexahedral elements is employed as the optimization objective function. The effectiveness and robustness of the approaches are demonstrated through two complex three-dimensional models.

1. Introduction

Finite element method plays more and more important roles in the field of the science research and engineering applications. An important requirement of this numerical method is to discretize the solid model into a ‘mesh’ composed of a number of elements. In three-dimensional numerical analysis, tetrahedron, hexahedron and a combination of them are usually used. Tetrahedral element meshes have the advantage of high efficiency, easy to implementation, flexible for adaptive mesh generation and easy to realize the mesh regeneration. At present, the automatic generation technology of tetrahedral element meshes is fully mature, and it is employed extensively to handle complex geometries. However, hexahedral element meshes have proven to be superior to tetrahedron element meshes in terms of analysis accuracy, number of elements, distortion resistance and regeneration times [1,2]. This makes hexahedra an attractive choice for the numerical analysis of three-dimensional problems.

Since automatic hexahedral mesh generation has a number of constraints, which are tied to the element shape itself as well as the connectivity requirements of the resulting mesh [3], none of the existing methods have proven to be an all-encompassing algorithm and each has drawbacks to their use. The existing schemes can be classified as direct or indirect methods [4]. Indirect methods convert a tetrahedral element into four hexahedral elements or combine five or more tetrahedral elements into a hexahedral element. The disadvantage of this method is that a high number of irregular nodes are introduced into the mesh resulting in poor element quality. Direct methods provide a discretized solid with hexahedral element meshes by generating hexahedral element directly. Currently, there are mainly four distinct strategies proposed for all-hexahedral mesh generation, including mapping/sweeping method, auto block structuring methods, the extended advancing front technique and the grid-based method. Mapping/sweeping method [5,6] can generate high-quality structured hexahedral element mesh, but its application range for three-dimensional model shapes is relatively narrow. Auto block structuring methods [7–9] includes medial axis, recent frame field and volume parameterization methods. The main problem of this method is how to subdivide a complex solid object into topologically simple

* Corresponding author.
E-mail address: huang0539@hotmail.com (L. Huang).

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subregions automatically. The extended advancing front technique mainly includes plastering method and whisker weaving method. Although the plastering method [10,11] produces well-shaped elements on the boundaries of the solid models, it cannot guarantee all-hexahedral elements in the interior. Whisker weaving [12,13], which is based on the concept of the spatial twist continuum (STC), has achieved some success, but has not yet to prove itself as robust and reliable method for various examples. Grid-based method [14,15] can generate well-shaped hexahedral elements in the interior of the solid models, but poor quality elements at boundaries. In spite of the undesirable features, the grid-based method is relatively simple to implement and easy to realize the local refinement.

Taking into account the pros and cons of the above mentioned algorithms, the grid-based method has advantages of high automation and easy local refinement. It is especially fit for adaptive generation and regeneration of all-hexahedral element mesh. Many researchers have done many research works on grid-based all-hexahedral mesh generation and considerable progress has been achieved. Schneiders et al. [16–19] proposed an octree-based algorithm for hexahedral mesh generation. This method can realized the local field refinement of the solid models. Su et al. [20] proposed an algorithm to generate an all-hexahedral mesh of a multi-domain solid model using a hybrid grid-based approach. A splitting technique and a wedge insertion technique were employed to resolve cases of degenerated elements. Lee and Yang [21] introduced surface element layer (SEL) which inserted between the boundary elements and the boundary of the domain being aligned around the boundary. Park and Yang [22] developed a conformal hexahedral mesh refinement technique by an iterative process of inserting zero-thickness element layers (ZTL) and by an optimal distribution of mesh density. But the density of the meshes generated by using the above schemes is not well controlled and the number of elements may increase considerably. Ito et al. [23] applied an octree-based mesh generation method to biomedical applications. Although their approach can get reasonable-quality, geometry-adapted unstructured hexahedral meshes, they leave sharp geometrical features out of consideration.

Adaptive mesh generation is an important factor in improving the accuracy of numerical simulation analysis. Adaptive generation techniques of the mesh can be divided into two types. The first is the adaptive technique of the initial mesh generation based on the geometric features of the solid model. The second is the adaptive refinement and regeneration techniques of the mesh based on a posteriori error estimators [22,24,25]. The latter has been employed widely and its effectiveness of solution-based mesh generation was proved. So far, many researches have been done in developing geometry-adaptive approaches, some sort of mesh sizing has been generated and is integrated with the meshing process [26–30]. Unfortunately, none of them is perfect, especially for the generation of three-dimensional hexahedral element mesh. The existing problems are mainly as follows: Firstly, they almost fail to take all the affecting factors into account when applying refinement, such as the geometry of the input solid, the topology requirement from an adjacent mesh, the physics of the problem, the user specification, etc. In this way, it is often difficult to describe the small geometric features of complex models, especially consider the local mesh refinement caused by the physical characteristics (such as the effective stress and effective strain rate concentration areas) in the process of finite simulations. These problems will directly influence the efficiency of simulation and accuracy of analysis. Secondly, how to make the surface and boundary of the hexahedral meshes well fit the surface and boundary of the solid model is also an important problem to be solved presently. Thirdly, due to the poor quality of the elements in the surface layer generated by the grid-based method, it is another challenging problem how to improve the quality of surface layer elements.

Aiming at solving the above problems, this paper put forward an improved grid-based mesh generation and regeneration algorithm to adaptively create hexahedral element meshes with reasonable mesh quality. A method for the mesh density generation and control was also presented. This method can automatically generate a reasonably optimized refinement field according to the surface curvatures, geometry features (including small and thin features), density windows and field variables distribution (such as stress, strain, strain rate and temperature, etc). Since one important requirement of mesh generation algorithm is that finite element meshes should give good description of the geometry features of a solid model, therefore, this paper summarized and gave eight different types of free element facet configurations for matching the surface of the hexahedral element meshes to the surface of the solid model. An optimal method for matching boundary between the hexahedral element meshes and the solid model was established by using the Jacobian-based approach. A precise matching of the hexahedral element meshes to the boundaries of the solid model was realized by using the above methods or approaches and according to the relative position of boundary elements and the characteristic boundaries of the solid model. By using the *Scaled Jacobian* and the *Condition Number* of Jacobian matrix as mesh quality evaluation criterion, this paper constructed a quality metric of hexahedral element mesh. A curvature-based Laplacian smoothing method for the elements on the characteristic boundaries was also proposed. It not only can improve the boundary elements quality but also preserve the geometrical characters after optimization. By taking the *Condition Number* of the Jacobian matrix as the objective function, an approach for improving the quality of the surface and interior nodes was proposed. After the optimization, the quality of the surface mesh of solid models was improved significantly. The whole mesh quality can effectively preserve the geometrical characters and satisfy the requirement of three-dimensional finite element numerical simulation.

2. Improved hexahedral mesh generation and regeneration algorithm

As mentioned above, the hexahedral mesh generation and regeneration algorithm developed in this paper is based on an improved grid-based method. Similar to the conventional gird-based method, an initial gird structure which is completely superposed on the solid model is generated. Unlike using a simple mapping algorithm to mesh a volumetric bounding box and unlike using a uniform size of the whole hexahedral elements, an adaptive generation algorithm was employed to create the initial gird structure based on the mesh density distribution. Fig. 1 shows the flow chart of adaptive generation for hexahedral mesh. Detailed explanations of the key techniques shown in the flow chart will be systematically presented in the following sections.

2.1. Solid model construction and boundary identification

For the initial mesh generation, it is necessary to construct a solid model, which can define its geometric features. In this paper, triangulated boundary representations generated by CAD systems, as stereo lithography (STL) files for example, are used. The content of STL files is the data information of a series of triangle patches that approach the surfaces of three-dimensional solid model, including the normal vector and the coordinates of three vertices of each triangle facet. Each triangle facet of the geometry definition is associated with the local surface curvature, which satisfies the requirement of the geometry-adaptive algorithm. Fig. 2 shows the procedures of the adaptive hexahedral element mesh generation of