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Isogeometric collocation for phase-field fracture models

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Highlights

- Isogeometric collocation can significantly speed up phase-field fracture computations.
- We advocate a hybrid collocation–Galerkin formulation.
- It handles Neumann boundary and multi-patch conditions, and higher-order boundary terms.
- The adaptive Galerkin resolution of the fracture zone is crucial for accuracy and efficiency.

Abstract

Phase-field models based on the variational formulation for brittle fracture have recently been shown capable of accurately and robustly predicting complex crack behavior. Their numerical implementation requires costly operations at the quadrature point level, which may include finding eigenvalues and forming tensor projection operators. We explore the application of isogeometric collocation methods for the discretization of second-order and fourth-order phase-field fracture models. We show that a switch from isogeometric Galerkin to isogeometric collocation methods has the potential to significantly speed up phase-field fracture computations due to a reduction of point evaluations. We advocate a hybrid collocation–Galerkin formulation that provides a consistent way of weakly enforcing Neumann boundary conditions and multi-patch interface constraints, is able to handle the multiple boundary integral terms that arise from the weighted residual formulation, and offers the flexibility to adaptively improve the crack resolution in the fracture zone. We present numerical examples in one and two dimensions that illustrate the advantages of our approach.

Keywords: Phase-field modeling; Fracture; Isogeometric collocation

1. Introduction

Phase-field fracture models introduced by Bourdin et al. [1] are based on the variational approach to brittle fracture due to Francfort and Marigo [2] where the solution to the fracture problem is found as the minimizer of a global

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energy functional. The basic idea of phase-field fracture is to represent cracks by a continuous scalar field that has a value of one away from the crack and is zero at the crack location. The phase-field serves as a multiplication factor to tensile energy components such that it locally penalizes the capability of the material to carry tensile stress at the crack location. In this sense, the phase-field idea is conceptually very similar to the fictitious domain approach applied e.g. in the finite cell method [3–8]. The diffusiveness of the crack approximation, i.e. the local slope of the phase-field at the crack location, is controlled by a length-scale parameter. From a mathematical point of view, the concept of Γ-convergence can be used to show that the phase-field model converges to the correct minimizing solution as the length-scale parameter goes to zero [9–11].

From a numerical point of view, phase-field fracture models open the door for computational analysis of complex crack patterns in three dimensions. While many established approaches that rely on the introduction of discontinuities into the displacement field by remeshing or by enriching basis functions (see for example [12–18]) have shown much success in modeling two-dimensional fracture problems, these methods have proven difficult to be extended to three-dimensional problems. The diffusive approximation of the crack by a continuous phase-field does not require the introduction of discontinuities. Cracks can be represented independently from the mesh and its topology by the solution of an additional differential equation. Moreover, the reformulation of the fracture problem as a system of partial differential equations that completely determine the evolution of cracks eliminates the need for any ad hoc rules or conditions to determine crack nucleation, propagation, or bifurcation. The phase-field fracture approach has proven to accurately and robustly capture crack behavior in two and three dimensions for quasi-static fracture [19–23], dynamic crack propagation [21,24–28], at finite strains [29], for fracture in piezo- and ferroelectric materials [30–32] and for cohesive fracture [33,34].

Isogeometric analysis (IGA) was introduced by Hughes and coworkers [35,36] to bridge the gap between computer aided geometric design (CAD) and finite element analysis. Its core idea is to use the same *smooth* and *higher-order* basis functions, e.g. non-uniform rational B-splines (NURBS), for the representation of both the geometry in CAD and the approximation of solution fields in analysis. The initial motivation for IGA was to simplify the cost-intensive mesh generation process required for standard finite elements and to support a more tightly connected interaction between CAD and finite element tools. Since then IGA has developed into an innovative computational mechanics technology, offering a range of new perspectives and opportunities that go far beyond the geometric point of view of analysis. One such opportunity is isogeometric collocation [37,38], which emerged from the search for more efficient quadrature rules for spline discretizations [39–41]. In contrast to Galerkin-type formulations, collocation is based on the discretization of the strong form of the underlying partial differential equations, which requires basis functions of sufficiently high order and smoothness. Consequently, the use of IGA for collocation suggests itself, since spline functions can be readily adjusted to any order of polynomial degree and continuity required by the differential operators at hand. The major advantage of isogeometric collocation is the minimization of the computational effort with respect to quadrature, since for each degree of freedom only one point evaluation at a so-called collocation point is required. This constitutes a significant advantage for applications where the efficiency and success of an analysis technology is directly related to the cost of quadrature. Isogeometric collocation has been applied successfully in elasticity and explicit elastodynamics [42], for the development of locking-free structural elements [43,44], in contact and plasticity problems [45], and for the Cahn–Hilliard phase-field model [46].

In this paper, we start to explore the application of isogeometric collocation to phase-field models of brittle fracture, considering the second-order phase-field formulations developed by Miehe, Welschinger and Hofacker [23,26] and Borden et al. [21], and the fourth-order phase-field formulation recently introduced by Borden et al. [24]. In addition to common quadrature point operations such as evaluating basis functions or computing matrix–matrix products, their numerical implementation requires a range of costly operations at the quadrature point level, such as finding eigenvalues, taking derivatives of tensor-valued functions, and forming tensor projection operators. Therefore, the formation and assembly of stiffness matrices constitutes a significant portion of the total computational cost. So far Galerkin methods have been the dominant tool for the discretization of phase-field fracture models, which require more than one quadrature point per basis function. In particular for higher-order isogeometric Galerkin discretizations, the number of quadrature point evaluations can be overwhelming (see e.g. [38]).

Isogeometric collocation has the potential to significantly speed up phase-field fracture computations by reducing the number of point evaluations. At the same time, it maintains the favorable approximation properties of higher-order smooth basis functions, which on a per-degree-of-freedom basis exhibit increased accuracy and robustness in comparison to standard C^0 basis functions [5,47,48] and superior spectral accuracy in the higher modes [49–51]. Moreover,