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An isogeometric collocation approach for Bernoulli–Euler beams and Kirchhoff plates

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Abstract

In this paper, IGA collocation methods are for the first time introduced for the solution of thin structural problems described by the Bernoulli–Euler beam and Kirchhoff plate models. In particular, a precise description of the proposed methods, of the relevant implementation details, and of the strategy to efficiently deal with different combinations of boundary conditions is given. Finally, several numerical experiments confirm that the proposed formulations represent an efficient and geometrically flexible tool for the simulation of thin structures.

Keywords: Collocation; Bernoulli–Euler beam; Kirchhoff plate; NURBS; Isogeometric analysis

1. Introduction

Isogeometric analysis (IGA) is a recently proposed computational technology, introduced by Hughes et al. [1,2] in 2005, with the main aim of bridging the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). The basic IGA paradigm consists of adopting the same basis functions used for geometry representations in CAD systems – such as, e.g., Non-Uniform Rational B-Splines (NURBS) – for the approximation of field variables, in an isoparametric fashion. This leads to a cost-saving simplification of the typically expensive mesh generation and refinement processes required by standard FEA, which was the original motivation for IGA. Moreover, thanks to the high-regularity properties of its basis functions, IGA has shown a better accuracy per-degree-of-freedom and an enhanced robustness with respect to standard FEA in a number of applications. Solids and structures [3–15] are a prime example, including also beam, plate and shell elements [16–20]. IGA has also been successful in fluid mechanics [21–27], and has opened the door to geometrically flexible discretizations of higher-order partial differential equations in primal form (see, e.g., [28–33,18]).

An important issue when dealing with IGA regards the development of efficient integration rules, able to reduce array formation costs in particular when higher-order approximations are employed. The fact that element-wise Gauss

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quadrature, typically used for standard FEA and originally adopted for Galerkin-based IGA, does not properly take into account the inter-element higher continuity of the IGA basis functions leads to sub-optimal array formation costs, which significantly affect the performance of IGA methods. *Ad-hoc* quadrature rules have been proposed in [34–36], but the development of a general and effective solution for Galerkin-based IGA methods is still an open problem.

Aiming at optimizing the computational cost, still taking advantage of IGA geometrical flexibility and accuracy, isogeometric collocation schemes have been recently proposed in [37,38]. The basic idea consists of the discretization of the governing partial differential equations in strong form, adopting the isoparametric paradigm and making use of the higher-continuity properties of the IGA shape functions. Detailed comparisons with both IGA and FEA Galerkin-based approaches have shown IGA collocation advantages in terms of accuracy versus computational cost, in particular when higher-order approximation degrees are adopted [39]. In general, IGA collocation features look particularly desirable in all those situations where evaluation and array formation costs are dominant. This is the case of explicit structural dynamics where the computational cost is dominated by stress divergence evaluations at quadrature points for the calculation of the residual force vector [38].

Within the IGA collocation context, several promising significant studies have been recently published, including phase-field modeling [40], contact [41], and hierarchical local refinement [39]. In particular, IGA collocation offers interesting possibilities in the framework of shear-deformable structural elements. Three-field (i.e., displacements, rotations, shear stresses) mixed formulations for Timoshenko initially-straight planar beams [42] and for curved spatial rods [43] have been successfully proposed. In such cases, the structure of IGA collocation leads to mixed methods which are locking-free independently of the approximation degrees for the three fields. Such a unique property has been proven analytically and extensively tested numerically. Following these positive results, isogeometric collocation has been then successfully applied also to the solution of Reissner–Mindlin plate problems, in both primal and mixed forms [44]. Finally, an interesting new single-parameter formulation for shear-deformable beams, recently introduced in [45], has been solved also via IGA collocation.

In the present paper, we introduce IGA collocation methods for the solution of thin structural problems described by Bernoulli–Euler beam and Kirchhoff plate models. We note that, although the use of IGA collocation for Bernoulli–Euler beams and Kirchhoff plates is new, collocation methods have a long history, in particular for the application to thin beams, plates, and shells. See, for example, [46–50]. Other related works fall into the category of the so-called differential quadrature methods [51–54]. A particular feature of Bernoulli–Euler beam and Kirchhoff plate models is that they are described by fourth-order differential equations, for which IGA collocation has already been shown to be an efficient and viable solution scheme in the context of Cahn–Hilliard phase-field modeling. With respect to [40], an enhanced strategy to efficiently deal with all relevant combinations of boundary conditions typical of plate problems is herein introduced. Several numerical experiments confirm the good behavior of the proposed formulations.

The paper is organized as follows. In Section 2, the simple model problem of a Bernoulli–Euler beam is considered. After the introduction of the boundary-value problem, the proposed numerical formulation based on IGA collocation is presented and numerically tested. In Section 3, Kirchhoff plates are considered. The boundary-value problem is stated and the adopted numerical formulation is discussed in detail, with special attention to the strategy for boundary condition imposition. Section 4 is then devoted to numerical tests showing the performance of the proposed collocation method for Kirchhoff plates, while, in Section 5, conclusions are drawn.

2. Bernoulli–Euler beams

We start illustrating the simple one-dimensional case represented by an initially straight Bernoulli–Euler beam problem. The differential equation governing the boundary-value problem is of fourth order and we solve it in strong form by means of an isogeometric collocation scheme. As we will point out later, the theoretical analysis provided in [37] for second-order problems can be easily extended to this case, guaranteeing the convergence of the method. In this section, we introduce the boundary-value problem in strong form, we describe the proposed numerical formulation based on isogeometric collocation, and we present numerical examples showing the good behavior of the method.

2.1. Boundary-value problem

The boundary-value problem associated with a Bernoulli–Euler beam can be stated as follows. Let $L > 0$ be the length of the beam and let us assume that $\Omega = (0, L)$ is the problem domain. The boundary of Ω is denoted by