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A flexible approach to model coupling through probabilistic pooling

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ABSTRACT

Model coupling is an important approach to studying the dynamics of complex systems, but by introducing new feedback loops, the dynamics of coupled models can be artificially distorted. This paper describes a new method of model coupling which addresses this problem through a dynamic form of regularization. The method allows the time series evolution of model variables to be mutually informed by multiple models, and models to influence each other in proportion to their degree of certainty. Uncoupled forms of the coupled models can act as dynamic priors on the trajectory of coupled variables, strengthening model stability and offering additional calibration of the coupling process. Finally, models that describe different spatial scales can be coupled into multi-scale models, so that, for example, spatially-distributed models can be coupled with aggregate models, and influence one another. We apply this technique to a coupled socio-ecological system of population growth and ecosystem harvesting.

Software availability

The model coupling system described below is implemented in the OpenWorld modeling framework, available at https://github.com/jrising/openworld/.

1. Introduction

A growing number of integrated models are being developed by combining existing models as components (Schlueter et al., 2012; Strasser et al., 2014). Combining existing models supports the development of more comprehensive models while building on the confidence placed in simpler models. Often, model components are linked together in a serial fashion, such as when a climate model is used to drive a economic or ecosystem model to study climate impacts. However, in many cases there are feedbacks between the model components, where the output of one model is an input to the models which determine its own input. For example, the impacts of climate on human and natural systems can result in changes in the release of CO_2 into the atmosphere. In this case, the component models are coupled (Brandmeyer and Karimi, 2000).

We define coupling as the introduction of feedback loops between two or more models, where some of the variables taken to be exogenously determined by each model are computed at each time-step by other models. Coupled models are used to study climate (Eyring et al., 2015), ecosystems (Lehodey, 2005; Fulton, 2010), earth systems (Sokolov et al., 2005; Warner et al., 2008), climate mitigation (Anthoff and Tol, 2010), the water-energy-food nexus (Hermann et al., 2011), and many other realms (Larson et al., 2005; Malleron et al., 2011). Coupled natural and human dynamics are of particular interest in understanding ecological and economic sustainability (Liu et al., 2007).

Coupling models can introduce dynamics which are not present in either model individually. While this is exactly the goal of model coupling, the impacts of this feedback can be distortive. In the worst case, the coupled system can have resonant frequencies which produce instability and run-away feedback, a problem which is sometimes resolved by increasing the strength of negative feedback drivers to unrealistic levels.

Even when runaway feedback does not occur, feedback can result in amplifications to the dynamics of the system. In some cases, this feedback is a accurate reflection of the true dynamics of the system (e.g., Roe and Baker, 2007). However, if the uncoupled original models were calibrated to reproduce observations, they typically require recalibration in their coupled form. This recalibration can be time consuming, disruptive to the scientific processes that underlay the uncoupled models, and can produce parameter estimates that are not realistic from the standpoint of the theories behind the individual uncoupled models. Often, a larger set of feedbacks could be formed between conceptual model components than is technically feasible, because of how the variables are defined or exposed to model developers. In all of these cases, it is desirable for the coupled model to take on some new behaviors but not to drift far from the behavior of the component

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models in their uncoupled form.

While the disruptive effects of feedback are well-known, modelling frameworks have traditionally focused on the mechanics of connecting the models and moving data between them (Larson et al., 2005; Fritzinger et al., 2012), rather than the distortions that can arise from this process. This paper presents a new technique for managing those distortions. Ultimately, the goal of this technique is to allow any collection of models to be safely coupled without recalibration and with the flexibility for models to be added and removed with minimal effort.

The paper is organized as follows. Section 1 formalizes the feedback problem. Section 2 presents the basic solution developed in this paper. Section 3 then extends this technique to a cross-scale context. The key parameter introduced by this technique to the modelling process is the strength of dynamic priors, which section 4 describes how to balance the effects of feedback and regularization. Finally, section 5 develops an example application of a socio-ecological system, and shows how its feedback distortions can be resolved and the spatial evolution of its dynamics can be explored through cross-scale coupling.

2. Coupled model feedback

We define a component model as a relationship between a set of state variables across time t, $\{x_i(t)\}$, and response variables also across time t, $\{y_j(t)\}$. The model may also have memory, through internal variables, $\{z_k(t)\}$. Furthermore, to support model coupling, we assume that the model is computed sequentially through time without access to future data. Finally, let the model have a set of parameters, $\{\theta_l\}$, used to calibrate it. Concretely, this means that

$$z_k(t) = g_k(t, \{x_i(s) \text{ for } s \le t\}, \{z_k(s) \text{ for } s < t\})$$

$$y_j(t) = h_j(t, \{z_k(s) \text{ for } s \le t\}, \{\theta_l\})$$

Here, $g_k(\cdot)$ and $h_j(\cdot)$ determine the evolution of z_k and y_j, respectively, through time. In this formulation, all input variables x_i are mediated through internal variables z_k, perhaps through a 1-to-1 mapping.

A model can then be represented as a tuple of relationships,

$$H = (\{g_k(X,Z)\}, \{h_j(Z,\Theta)\})$$

Coupling consists of a collection of functions between the variables in a set of models, $\{H_m\}$. The coupling could occur between the input and output variables, but it may also intervene in variables that are otherwise internal. Either of the following are possible relationships between model m and model n:

$$x_{mi}(t) = a_{mni}\left(\left\{y_{nj}(t)\right\}, \{z_{nk}(t)\}\right)$$

$$z_{mk}(t) = b_{mnk}\left(\{x_{mi}(t)\}, \{z_{mk}(t)\}, \left\{y_{nj}(t)\right\}, \{z_{nk}(t)\}\right)$$

$a_{mni}(\cdot)$ then describes how variables in model n determine the ith input to model m, and $b_{mnk}(\cdot)$ does the same for the kth internal variable of model m.

Feedback in coupled systems produces two common distortive effects: runaway feedback and miscalibration. Runaway feedback manifests as exponentially amplifying trajectories or oscillations which are produced by neither model in isolation but result from the coupled system. Miscalibration occurs when coupling causes the dynamics of a given $y_j(t)$ to change, when it should not be affected by the feedback.

To understand these issues, consider a system composed of two coupled models, H_1 and H_2. Let the models be represented as linear, time-invariant systems (LTI), characterized by transfer functions $h_1(t)$ and $h_2(t)$, respectively. These models take a single time series of input and provide a time series of output. The analytical results in this paper all apply LTI theory, although the final application of this paper is formulated for any nonlinear system conforming to the description above (Oppenheim et al., 2014).

A wide variety of models can be represented as LTI systems, including dynamical models consisting of stocks and flows (e.g., Sterman, 2001). The output $y_m(t)$ that results from passing the input $x_m(t)$ into a single system H_m is computed, $y_m(t) = h_m(t)*x(t)$, where $*$ is the convolution operator. Although the structure only allows a single input and output to each model, multiple inputs and outputs can be interleaved into a single time series, and allowed to interact through a careful definition of the transfer function.

In their uncoupled form, let model H_m take as its input $x_m(t)$ and produce $y_m(t)$. Furthermore, let $y_1(t)$ and $x_2(t)$ refer to the same physical quantity, providing one path for coupling, and let $y_2(t)$ represent a computed anomaly on a physical quantity represented by $x(t)$, and produce the model input $x_1(t) = x(t) + y_2(t)$. For example, $x(t)$ might be an available food supply, H_1 a species ecosystem growth relationship, and $y_1(t)$ the population of that species. This population is an input into H_2, which then computes a loss of food supplies due to crowding, which compound the normal food-based carrying capacity already reflected in H_1. These definitions are laid out below.

Model	Input	(Example)	Output	(Example)
H_1	$x(t)+y_2(t)$	(food availability)	$y_1(t)$	(species population)
H_2	$y_1(t)$	(species population)	$y_2(t)$	(crowding impacts)

In Fig. 1, we couple these systems with a simple feedback loop. $x(t)$ is exogenous, but the feedback loop contributes to determining the evolution of $y_1(t)$.

This relationship is described as

$$y_1(t) = h_1(t)*(x(t) + y_2(t))$$

$$y_2(t) = h_2(t)*y_1(t)$$

Under a Laplace transformation ($\mathscr{L}(x(t)) = X(s)$, $\mathscr{L}(y_1(t)) = Y(s)$ and so on), this can be written

$$Y_1(s) = H_1(s)(X(s) + Y_2(s))$$

$$Y_2(t) = H_2(s)Y_1(s)$$

which simplifies to

$$Y_1(s) = \frac{H_1(s)}{1 - H_1(s)H_2(s)}X(s) = \frac{H_1(s)}{1 - F(s)}X(s)$$

Here, $F(s) = H_1(s)H_2(s)$ is the feedback term. If $F(s) > 1$ for any s, the system will resonate at this complex frequency and produce runaway feedback.

For the miscalibration effect, we suppose that two models are estimated separately before being coupled, producing $H'_1(s)$ from the relationship between $x(t)$ and $y_1(t)$ and $H'_2(s)$ from $y_1(t)$ and $y_2(t)$, shown on the right of Fig. 1. That is,