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Single-variable formulations and isogeometric discretizations for shear deformable beams

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Highlights

- We present Timoshenko beam formulations with only one unknown variable.
- We introduce the bending displacement as new variable.
- Strong and weak forms of the problem are developed.
- The problems are solved by isogeometric Galerkin and collocation formulations.
- The presented numerical methods are completely locking-free ab initio.

Abstract

We present numerical formulations of Timoshenko beams with only one unknown, the bending displacement, and it is shown that all variables of the beam problem can be expressed in terms of it and its derivatives. We develop strong and weak forms of the problem. The strong form of the problem involves the fourth derivative of the bending displacement, whereas the symmetric weak form involves, somewhat surprisingly, third and second derivatives. Based on these, we develop isogeometric collocation and Galerkin formulations, that are completely locking-free and involve only half the degrees of freedom compared to standard Timoshenko beam formulations. Several numerical tests are presented to demonstrate the performance of the proposed formulations.
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1. Introduction

The two major theories for structural analysis of beams are the Bernoulli–Euler and Timoshenko theories. In the Bernoulli–Euler theory cross sections are assumed to remain straight and normal to the mid-axis during deformation, which implies that the rotation of a cross section can be obtained from the derivative of the deflection. This assumption corresponds to neglecting shear deformation and is valid for thin beams. In the Timoshenko theory cross sections

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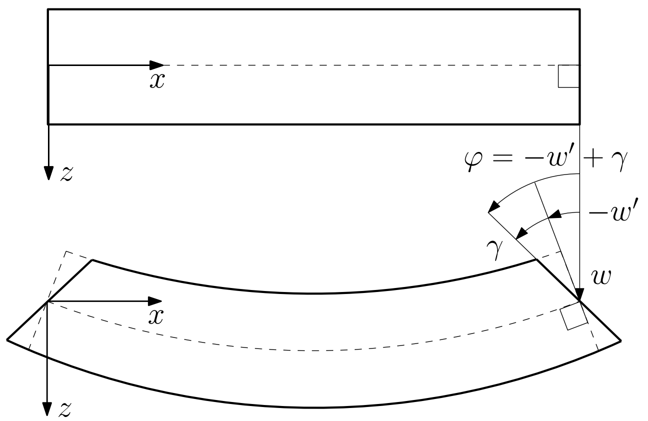

Fig. 1. Beam model and kinematic variables.

are also assumed to remain straight but not necessarily normal to the mid-axis, which accommodates shear deformation [1]. As a consequence, rotation and deflection are typically considered independent variables. In *Thomas et al.* [2] an overview of different classical Timoshenko beam finite elements can be found.

In this paper, we present a relation between deflection and rotation for Timoshenko beams which leads to formulations with only one variable. In particular, we split the displacement into a bending and a shear part, and show that these two parts are related, which allows expressing all derived variables, such as rotation and shear strain, in terms of the bending displacement. The idea of splitting the displacement of Timoshenko beams into a bending and a shear part can be found in papers from the early days of beam finite elements; see, for example, *Kapur* [3]. In [3], however, no relation between the bending and shear part is established. Instead, displacement and rotation are discretized independently for both the bending and shear part, resulting in four unknown fields, which eventually yields a formulation with twice the number of degrees of freedom compared to a standard formulation.

In the present paper, we derive a single differential equation for the Timoshenko beam problem with the bending displacement as the only unknown. This differential equation is form-identical to the one for a Bernoulli–Euler beam model but fully accounts for shear deformation. Similar approaches have been presented by *Li* [4] and *Falsone et al.* [5]. In both papers, the Timoshenko beam problem is reduced to a single equation in terms of one variable. These approaches do not utilize the split into bending and shear terms, but the resulting equations are essentially equivalent to the one presented in this paper. However, *Li* [4] does not develop a weak formulation and does not pursue a discrete formulation, or numerical calculations. *Falsone et al.* [5] develop a weak form and a corresponding finite element formulation, but their weak form is not symmetric and produces boundary terms at element level, which need to be coupled between elements. Furthermore, an additional change of basis at element boundaries is required in order to guarantee continuity of the displacement between elements. In the present paper, we establish a weak form which is both symmetric and without boundary terms at element level, but involves both second and third derivatives of the bending displacement. Due to the isogeometric basis functions adopted in this paper, suitable degree and continuity of the bending displacement are automatically guaranteed. In addition to the Galerkin formulation, we develop an isogeometric collocation method which utilizes the strong form of the problem. Since there is only one unknown variable, the formulations employ half the number of degrees of freedom compared to standard Timoshenko beam formulations, and, at the same time, are completely locking-free by construction. Numerical examples demonstrate the validity and efficiency of the presented methods. In the authors' view, the formulation provides a new paradigm for the development of bending elements for structural analysis.

2. Governing equations of the Timoshenko beam model

We consider a straight beam with planar deformation, as depicted in Fig. 1. Cross sections are assumed to remain straight during deformation, but not necessarily perpendicular to the beam axis due to shear deformability. The transverse displacement of a cross section is denoted by w, the total rotation by φ, and the shear deformation by γ. All variables are functions of the coordinate x and a prime symbol $(\cdot)'$ indicates a derivative with respect to x, i.e., $(\cdot)' = \mathrm{d}(\cdot)/\mathrm{d}x$.

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