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An immersogeometric variational framework for fluid–structure interaction: Application to bioprosthetic heart valves

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Abstract

In this paper, we develop a geometrically flexible technique for computational fluid–structure interaction (FSI). The motivating application is the simulation of tri-leaflet bioprosthetic heart valve function over the complete cardiac cycle. Due to the complex motion of the heart valve leaflets, the fluid domain undergoes large deformations, including changes of topology. The proposed method directly analyzes a spline-based surface representation of the structure by immersing it into a non-boundary-fitted discretization of the surrounding fluid domain. This places our method within an emerging class of computational techniques that aim to capture *geometry* on non-boundary-fitted analysis meshes. We introduce the term "immersogeometric analysis" to identify this paradigm.

The framework starts with an augmented Lagrangian formulation for FSI that enforces kinematic constraints with a combination of Lagrange multipliers and penalty forces. For immersed volumetric objects, we formally eliminate the multiplier field by substituting a fluid–structure interface traction, arriving at Nitsche's method for enforcing Dirichlet boundary conditions on object surfaces. For immersed thin shell structures modeled geometrically as surfaces, the tractions from opposite sides cancel due to the continuity of the background fluid solution space, leaving a penalty method. Application to a bioprosthetic heart valve, where there is a large pressure jump across the leaflets, reveals shortcomings of the penalty approach. To counteract steep pressure gradients through the structure without the conditioning problems that accompany strong penalty forces, we resurrect the Lagrange multiplier field. Further, since the fluid discretization is not tailored to the structure geometry, there is a significant error in the approximation of pressure discontinuities across the shell. This error becomes especially troublesome in residual-based stabilized methods for incompressible flow, leading to problematic compressibility at practical levels of refinement. We modify existing stabilized methods to improve performance.

To evaluate the accuracy of the proposed methods, we test them on benchmark problems and compare the results with those of established boundary-fitted techniques. Finally, we simulate the coupling of the bioprosthetic heart valve and the surrounding blood flow under physiological conditions, demonstrating the effectiveness of the proposed techniques in practical computations.

Keywords: Fluid–structure interaction; Bioprosthetic heart valve; Immersogeometric analysis; Isogeometric analysis; Nitsche's method; Weakly enforced boundary conditions

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1. Introduction

Heart valves are passive structures that open and close in response to hemodynamic forces, ensuring proper unidirectional blood flow through the heart. At least 280,000 diseased heart valves are surgically replaced annually [1,2]. By far the most popular surgical replacements are the bioprosthetic heart valves (BHV), which are fabricated from biologically derived materials, with the design goal of mechanical similarity to native valves. Like native valves, BHVs are composed of thin flexible leaflets that are pushed open by blood flow in one direction and closed by flow in the other direction. BHVs have more natural hemodynamics than the older "mechanical" prostheses designs, which are comprised of rigid leaflets and require life-long anticoagulation therapy [2]. However, the durability of a typical BHV remains limited to about 10–15 years, with failure resulting from structural deterioration, mediated by fatigue and tissue mineralization [1–3]. While much effort has gone into developing methods to mitigate mineralization, methods to extend durability remain largely unexplored. A critical part of such efforts to improve the design of BHVs is understanding the stresses acting on leaflets over the complete cardiac cycle.

Some previous computational studies on heart valve mechanics have used (quasi-)static [4,5] and dynamic [6] structural analysis, with assumed pressure loads on the leaflets. This produces deformation and stress distributions that can be used to understand the mechanical behavior of BHVs. However, the assumed pressure load only crudely approximates the interaction between blood and valvular structures. A purely structural analysis is only applicable to static pressurization of a closed valve, which represents only a portion of the full cardiac cycle. It is therefore important to develop a computational framework that is able to simulate the dynamics of heart valves interacting with hemodynamics – a method for computational fluid–structure interaction (FSI) – which considers the complete mechanical environment of the valve and applies more accurate tractions to the leaflets during the entire cardiac cycle.

Many FSI methods employ boundary-fitted approaches, where the fluid problem is solved on a mesh that deforms around a Lagrangian structure mesh, matching it at the shared interface. The fluid problem on the deforming domain is said to be posed in an arbitrary Lagrangian–Eulerian (ALE) coordinate system [7–9]. In the FSI literature, the term ALE is sometimes reserved for numerical methods using finite elements in space and finite differences in time, distinguishing them from methods that use space–time finite elements, such as the deforming-spatial-domain/stabilized space–time (DSD/SST) technique [10,11]. Boundary-fitted FSI methods have been applied to challenging classes of real-world problems, including cardiovascular [12–17], parachute [18–24], and wind turbine [25–27] applications. The history, state-of-the-art, and practical applications of ALE and DSD/SST methods for FSI are covered thoroughly by Bazilevs et al. [28]. Boundary-fitted methods have the advantage of satisfying kinematic constraints by construction but, for scenarios that involve large translational and/or rotational structural motions, the boundary-fitted fluid mesh can become severely distorted if it is continuously deformed from a single reference configuration, harming both the conditioning of the discrete problem and the accuracy of its solution.

Applying boundary-fitted methods to complex engineered systems may therefore require specialized solution strategies to maintain fluid mesh quality. One approach is remeshing, in which all or part of the fluid domain is automatically re-discretized in space when mesh distortion becomes too extreme [29–32]. Mesh management is complicated further if the structure moves into and out of contact with itself, changing the topology of the fluid domain. For some applications, it may be sufficient to use specialized contact algorithms that modify the problem to enforce a small minimum separation between surfaces that would otherwise come into contact [33]. In our application to a heart valve, however, the ability of the structure to close and block flow is an essential aspect of the problem. Recent work [34,35] has extended DSD/SST methods to include true changes of topology without remeshing, but has so far only been applied to problems in which the boundary motion is known beforehand and prescribed. While the rigid motions of hinged mechanical prosthetic heart valves have been successfully studied with boundary-fitted methods [36,37], it is our opinion that maintaining mesh quality would become prohibitively difficult in a boundary-fitted simulation of a native or bioprosthetic heart valve, where flexible leaflets deform and contact each other in complex patterns that cannot be parameterized by a small set of variables.

For these reasons, non-boundary-fitted approaches have become a popular alternative for computational FSI [38–44], and are the focus of the present contribution. The first non-boundary-fitted approach to become widely known for computational fluid dynamics (CFD) was Peskin's immersed boundary method [45,46]. In non-boundary-fitted methods, a separate structural discretization is arbitrarily superimposed onto (or *immersed* into) a background fluid mesh. Such methods are particularly attractive for applications with complex moving boundaries, because they alleviate the difficulties of deforming the fluid mesh. Non-boundary-fitted methods can also handle change of fluid do-