



# Isogeometric analysis of sound propagation through laminar flow in 2-dimensional ducts

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## Highlights

- We model the propagation of sound through a slowly moving fluid in 2D ducts.
- B-splines are used to represent the geometry and approximate the flow and sound.
- A mode matching formulation is used for acoustic truncation boundary conditions.
- High order B-spline representations yield good approximations of acoustic duct modes.
- We find a strong sensitivity of sound to flow for a certain geometry and frequency.

## Abstract

We consider the propagation of sound through a slowly moving fluid in a 2-dimensional duct. A detailed description of a flow-acoustic model of the problem using B-spline based isogeometric analysis is given. The model couples the non-linear, steady-state, incompressible Navier–Stokes equation in the laminar regime for the flow field, to a linear, time-harmonic acoustic equation in the low Mach number regime for the sound signal. B-splines are used both to represent the duct geometry and to approximate the flow and sound fields. This facilitates an exact representation of complex duct geometries, as well as high continuity approximations of state variables. Acoustic boundary conditions on artificial truncation boundaries are treated using a mode matching formulation. We validate the model against known acoustic modes for a uniform flow through a straight duct. Improved error convergence rates are found when the acoustic pressure is approximated by higher order polynomials. Based on the model, we examine how the acoustic signal varies with sound frequency, flow speed and duct geometry. A combination of duct geometry and sound frequency is identified for which the acoustic signal is particularly sensitive to the flow speed.

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## 1. Introduction

It is well known in physics and engineering applications that wave propagation in ducts is sensitive to the duct geometry [1–3]. In particular, spatial resonances exist at certain frequencies [4–6], and it is anticipated that disturbances may cause large variations in signal transmission at such frequencies. While these effects have been explored in quantum mechanics, solid state physics, and optics, in particular for quasi-periodic and periodic structures [7–11], we here investigate the influence of a slow background flow on acoustic wave propagation in 2-dimensional ducts of varying height.

Acoustic wave propagation through flows in inhomogeneous ducts has been studied extensively in the literature, based on both numerical and analytical approaches, see e.g. [12–14]. Finite element methods represent a popular numerical approach, based on, e.g., the convected Helmholtz equation [15,16], the linearized Euler equations [17], or the so-called Galbrun's equation [18–20]. These methods are often discretized using Lagrange elements with  $C^0$ -continuity of the state variable approximations.

The current work contributes to the field of numerical methods for sound propagation through flow in ducts in two ways. Firstly, we construct a simple one-way coupled flow-acoustic model, where “one-way coupled” reflects that the flow field affects the acoustic field, but not vice versa. This is a good approximation for low-intensity acoustic excitations. The procedure we follow is first to compute the background flow based on the steady-state, incompressible Navier–Stokes equations in the laminar regime, i.e., at low Reynolds numbers, and then adopt a linear, time-harmonic flow-acoustic approach for the sound wave propagation in the low Mach number regime, using the output of the flow model as input to the acoustic model. This results in a single equation in the acoustic pressure, linear in both the flow field itself and its gradient. The low Mach number assumption corresponds to neglecting quadratic and higher order terms in the flow for the acoustic analysis.

Secondly, we base our calculations on isogeometric analysis as numerical method. This unites the powers of finite element methods to solve partial differential equations with the powers of computer aided design (CAD) to represent complex shapes [21,22]. A key feature of this approach is to use (variations of) B-splines, the building blocks of many CAD models, both to represent geometry and to approximate state variables. This allows for simple descriptions of complex duct geometries without any approximations, and provides high degrees of smoothness for both flow and acoustic fields with attractive error convergence properties. Another important feature of the method is that the acoustic eigenvalues and modes that enter the boundary conditions on artificial truncation boundaries in the proposed mode matching formulation, are represented very well with B-splines [23]. Through a consistent choice of B-splines to approximate the acoustic modes on the boundary and the acoustic field on the interior, a smooth and seamless model is obtained that requires no specification of parameters, like the absorption coefficient needed in the perfectly matched layer (PML) formulation. The applicability of isogeometric analysis is well documented for fluid mechanics, see, e.g., [24–27], wave phenomena and structural vibrations [23,28,29], shock hydrodynamics [30], and acoustics based on the boundary element method [31].

The first aim of the paper is to establish and validate the proposed flow-acoustic model for time-harmonic sound propagation at low Mach numbers through a stationary, incompressible background flow at low Reynolds numbers in 2-dimensional ducts using B-spline based isogeometric analysis. The second aim of the paper is to use the model to numerically examine how the geometry of the duct influences the sound propagation, and in particular its effect on how the acoustic signal depends on flow speed. The generic problem is sketched in Fig. 1. We consider a transmitting ultrasonic transducer mounted on the wall of a narrow duct in which we have a laminar flow of, say, air, and we examine how the acoustic signal depends on duct geometry, sound frequency, and flow speed. The exact representation of complex shapes in few degrees of freedom, and the smooth approximations of state variables embedded in the isogeometric method, make it ideal in this context.

The outline of the rest of the paper is as follows: Section 2 introduces the governing equations and approximations leading to the flow-acoustic model. The isogeometric method is presented in Section 3, after which the method is validated in Section 4 based on acoustic duct modes. Numerical results are presented in Section 5, and finally conclusions are summarized in Section 6.

## 2. Governing equations and approximations

We consider the propagation of sound through a moving fluid in a symmetric 2-dimensional duct as depicted in Fig. 1. The state of the fluid is characterized by the velocity  $\mathbf{u} = (u, v)$ , the pressure  $p$ , and the density  $\rho$ . These state

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