Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 285 (2015) 32–63
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Assessment of variational multiscale models for the large eddy simulation of turbulent incompressible flows

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Received 19 November 2013; received in revised form 24 October 2014; accepted 25 October 2014
Available online 11 November 2014

Abstract

In this work we study the performance of some variational multiscale models (VMS) in the large eddy simulation (LES) of turbulent flows. We consider VMS models obtained by different subgrid scale approximations which include either static or dynamic subscales, linear or nonlinear multiscale splitting, and different choices of the subscale space. After a brief review of these models, we discuss some implementation aspects particularly relevant to the simulation of turbulent flows, namely the use of a skew symmetric form of the convective term and the computation of projections when orthogonal subscales are used. We analyze the energy conservation (and numerical dissipation) of the alternative VMS formulations, which is numerically evaluated. In the numerical study, we have considered three well known problems: the decay of homogeneous isotropic turbulence, the Taylor–Green vortex problem and the turbulent flow in a channel. We compare the results obtained using different VMS models, paying special attention to the effect of using orthogonal subscale spaces. The VMS results are also compared against classical LES scheme based on filtering and the dynamic Smagorinsky closure. Altogether, our results show the tremendous potential of VMS for the numerical simulation of turbulence. Further, we study the sensitivity of VMS to the algorithmic constants and analyze the behavior in the small time step limit. We have also carried out a computational cost comparison of the different formulations. Out of these experiments, we can state that the numerical results obtained with the different VMS formulations (as far as they converge) are quite similar. However, some choices are prone to instabilities and the results obtained in terms of computational cost are certainly different. The dynamic orthogonal subscales model turns out to be best in terms of efficiency and robustness.

Keywords: Turbulence; Large eddy simulation; Stabilization; Variational multiscale

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http://dx.doi.org/10.1016/j.cma.2014.10.041
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1. Introduction

LES techniques for the numerical simulation of turbulent flows [1] are based on a scale separation that permits to reduce the computational cost with respect to direct numerical simulation (DNS). Such scale separation is traditionally achieved by filtering the original Navier–Stokes equations, which leads to an extra forcing term defined by a physical (functional or structural) model. This widely used approach is usually referred to as explicit LES [1].

By contrast, implicit LES techniques (ILES) rely on purely numerical artifacts without any modification of the continuous problem. This approach was seldom followed, the MILES (Monotone Integrated LES) approach [2–4] being the main exception, until the VMS method was introduced [5,6] and subsequently proposed as an ILES method (see below). ILES techniques are usually considered to be based on the addition of purely dissipative numerical terms, see [1, Section 5.3.4]. It is worth emphasizing that this is not the case of some particular VMS models, as it is shown in [7] and discussed below.

VMS was introduced in [5,6] as a framework for the motivation and development of stabilization techniques, which aim to overcome numerical difficulties encountered when using the standard Galerkin method. On the one hand, the velocity and pressure finite element (FE) spaces need to satisfy the *inf–sup* compatibility condition that guarantees pressure stability and precludes the use of equal order interpolation. Mixed methods satisfying this condition can be used and their finite volume counterpart, based on staggered grids, are common in the LES community. Stabilization techniques that permit the use of equal order interpolation were proposed, e.g., in [8,9]. On the other hand, global nonphysical oscillations appear in the convection dominated regime, when the mesh is not fine enough, that is, for high mesh Reynolds number flows. The only way to overcome this problem is through the addition of some form of dissipation which was recognized in the early development of stabilized methods [10]. Let us note that the common practice in the LES community is to rely on the explicit extra term introduced by the physical model using high order approximations of the convective term.[1]

The first attempts to perform LES using VMS concepts, presented in [11–15], were performed introducing explicit subgrid modeling. The VMS models used in these works split resolved scales into large and small, introducing an explicit LES model to account for the small scales stress tensor, e.g., a Smagorinsky-type dissipative term acting on the small scales only. As a result, an important fraction of the degrees of freedom is used for the small resolved scales whereas consistency is retained in the large resolved scales only.

ILES using a VMS approach with resolved and unresolved subgrid scales (the setting that permits to recover stabilized formulations) was suggested in [16] and performed in [17–19]. Excellent results were first presented in [18], but using isogeometric analysis for the space approximation [20]. Compared to classical LES based on filtering, the VMS approach does not face difficulties associated to inhomogeneous non-commutative filters in wall-bounded flows. Further, it retains numerical consistency in the FE equations and optimal convergence up to the interpolation order whereas, e.g., Smagorinsky models introduce a consistency error of order $h^{4/3}$ (see [11,12,18]).

Scale separation is achieved in the VMS formalism by a variational projection. The continuous unknown is split into a resolvable FE component and an unresolvable subgrid or subscale component. The action of the subscales onto the FE scales can be approximated in different ways, leading to different VMS models but in all cases these models are *residual based* (no eddy viscosity is introduced), which permits to retain consistency. Among the modeling possibilities is the choice of the subscale space, first discussed in [21], where it was enforced to be L^2-orthogonal to the FE space. Another modeling ingredient is the possibility of considering time-dependent subscales and to keep the VMS decomposition in all the nonlinear terms, which was studied in [16,22]. Clear improvements have been observed when using dynamic and fully nonlinear models for the simulation of laminar flows [22,23].

In this work we assess implicit VMS models for the numerical simulation of turbulent flows. We refer to the original references for a comprehensive treatment of the assumptions of the formulations and their numerical analysis. Our intention here is to compare the different VMS schemes in terms of quality of the results and computational cost, and discuss some implementation aspects that we find particularly relevant for the simulation of turbulent flows. Our main motivation is to compare the influence of using orthogonal subscales, in order to enrich current comparisons on VMS techniques for large-eddy simulations, such as [24], where only non-projected subscales are considered. We

[1] It is worth pointing out that both problems (convection instability and compatibility conditions) are also present in the *linear* Oseen problem. One of the inconsistencies of an explicit LES approach without a numerical dissipation term is that convection is stabilized by a term that comes from the physical model of the nonlinear Navier–Stokes equations and such a term is not present when the linear Oseen problem is considered.