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A semi-smooth Newton method for orthotropic plasticity and frictional contact at finite strains

Alexander Seitz, Alexander Popp*, Wolfgang A. Wall

Institute for Computational Mechanics, Technische Universität München, Boltzmannstrasse 15, D–85748 Garching b. München, Germany

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Highlights

- A novel combined FEM approach for contact and finite strain plasticity is developed.
- General isotropic hyperelasticity and orthotropic Hill plasticity are considered.
- All discrete inequalities are reformulated as semi-smooth complementarity functions.
- Efficient non-smooth versions of Newton's method handle all involved nonlinearities.
- The proposed semi-smooth Newton method is competitive to classical return mapping.

Abstract

A new approach for the unified treatment of frictional contact and orthotropic plasticity at finite strains using semi-smooth Newton methods is presented. The contact discretization is based on the well-known mortar finite element method using dual Lagrange multipliers to facilitate the handling of the additional Lagrange multiplier degrees of freedom. Exploiting the similarity of the typical inequality constraints of plasticity and friction, all involved discrete inequalities are reformulated as nonlinear non-smooth equations using complementarity functions. The resulting set of discrete semi-smooth equations can be solved efficiently by a variant of Newton's method, where all additionally introduced variables are condensed from the global system so that a linear system only consisting of the displacement degrees of freedom has to be solved in each iteration step. In contrast to classical radial return mapping methods for computational plasticity, the plastic constraints are not required to hold at every iterate in the nonlinear solution procedure, but only at convergence. This relaxation in the pre-asymptotic behavior results in an increased flexibility regarding algorithm design and a potentially higher robustness compared to radial return mapping algorithms. The presented elasto-plasticity algorithm includes arbitrary isotropic hyperelasticity, an anisotropic Hill-type yield function with isotropic and kinematic hardening, plastic spin and appropriate finite element technology for nearly incompressible materials. Therefore, it is well suited for the modeling of sheet metal forming and similar processes. Several numerical examples underline the robustness of the proposed plasticity algorithm and the efficient treatment of elasto-plastic contact problems.

Keywords: Finite strain plasticity; Anisotropic Hill model; Frictional contact; Mortar finite element methods; Semi-smooth Newton methods; Nonlinear complementarity functions

* Corresponding author.
E-mail address: popp@lnm.mw.tum.de (A. Popp).

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1. Introduction

In many engineering applications, ranging from forming processes to crash and impact scenarios, plasticity and contact in the context of finite deformations or even finite strains come hand in hand. This obviously necessitates efficient, robust and accurate discretization schemes as well as solution algorithms for elasto-plastic contact. In computational contact mechanics, mortar methods using Lagrange multipliers for the constraint enforcement have been of particular interest over the past decade, thanks to their sound variational foundation, see e.g. [1–5]. To achieve a localization of the individual discrete constraints and avoid an increase of the resulting system size by the additional Lagrange multipliers, so-called dual Lagrange multiplier interpolations have been introduced in the context of domain decomposition [6–10]. Due to a biorthogonality property, they allow for a trivial algebraic condensation of the additional Lagrange multiplier degrees of freedom. When applied to contact mechanics, these dual Lagrange multiplier methods have been efficiently combined with semi-smooth Newton methods as an active set strategy [11], see e.g. [12,13] for applications to small deformation frictionless and frictional contact and [14–19] for finite deformation frictionless and frictional contact. Within these approaches, the discrete inequality constraints of (frictional) contact are reformulated using semi-smooth nonlinear complementarity (NCP) functions. Consequently, the resulting formulations are well-suited for generalized Newton methods, see e.g. [20,21], and the contact nonlinearities can be treated within the same Newton type iteration as geometrical, material and other nonlinearities. For a profound review on state-of-the-art contact discretizations as well as solution algorithms the reader is referred to [22].

In computational plasticity, the vast majority of research papers focus on new material models rather than algorithms to solve the resulting constrained equations; with regard to solution algorithms radial return mapping methods have been the most common choice by far since the early days of computational plasticity and were extended to finite strains for the first time in [23,24]. Meanwhile, they can be found in all standard text books, e.g. [25,26]. Recently, based on the variational formulation of small strain plasticity, cf. [27], novel solution techniques for this case have emerged, namely sequential quadratic programming [28], interior point algorithms [29] and NCP functions [30,31] based on semi-smooth Newton methods. Due to the fundamentally different kinematic description of plasticity at finite strains compared to small strains (multiplicative vs. additive kinematics), however, these methods cannot be transferred directly to nonlinear kinematics. At finite strains, variational constitutive updates as proposed in [32] and more recently in [33–36] have been shown to offer improved efficiency compared to radial return mapping methods under certain conditions, see e.g. [36].

In this contribution, we present an NCP function based formulation of finite strain plasticity based on the work in [31] for small deformations. Our formulation not only covers von Mises plasticity, but also an orthotropic Hill-type yield criterion and an evolution of plastic spin [37]; two ingredients especially important for the application to sheet metal forming. Moreover, our formulation is based on the multiplicative decomposition of the deformation gradient into an elastic and a plastic part, and in contrast to most efficient radial return mapping methods using this multiplicative kinematics, it is not restricted to a certain form of hyperelastic material behavior, but naturally includes any isotropic hyperelastic relation in the elastic realm. Finally, in contrast to the previous NCP based plasticity formulations [30,31] using one scalar and one tensor-valued NCP function, we only use one tensor-valued NCP function, and thus reduce the number of additional unknowns at each quadrature point. These additional unknowns are eliminated from the global system of equations via element-local condensation such that the remaining linearized system to be solved only consists of displacement degrees of freedom, just like for the well-known radial return mapping algorithm. This novel approach for finite strain plasticity allows for an efficient and uniform treatment of elasto-plastic frictional contact.

The remainder of the paper is outlined as follows. In Section 2, we introduce the strong form of the initial boundary value problem of elasto-plastic frictional contact at finite strains. This includes the kinematics of frictional contact and finite strain plasticity. Section 3 introduces a finite element space discretization and an appropriate time discretization of the time dependent effects of friction and plasticity. In Section 4, we present the novel formulation of a semi-smooth Newton method for finite strain plasticity, including the consistent linearization of all involved terms. Section 5 shortly outlines the application of F-bar finite element technology to our plasticity formulation. This is crucial to deal with plastic incompressibility when using first-order finite elements to avoid spurious locking effects. Finally, several numerical examples in Section 6 demonstrate the robustness and accuracy of the plasticity algorithm and its efficient combination with a dual Lagrange multiplier contact formulation. We close with several concluding remarks and an Appendix on general isotropic hyperelasticity for multiplicative elasto-plastic kinematics.