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Extended embedded finite elements with continuous displacement jumps for the modeling of localized failure in solids

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Highlights

- Novel embedded finite elements with continuous displacement jumps are proposed.
- The enriched discontinuity modes satisfy *a priori* the traction continuity condition.
- General relative stretching modes are identified with no limit of a vanishing Poisson's ratio.
- The model is intrinsically stress locking free, numerically stable and of high coarse mesh resolution.
- Boundary conditions and linear equation constraints can be trivially imposed on the displacement jumps.

Abstract

This paper addresses novel extended embedded finite elements with continuous displacement jumps for the modeling of localized failure in solids. On one hand, based upon the unified multiscale kinematics of strong discontinuities, standard finite elements at the coarse scale are enriched with fine scale kinematics in which non-uniform discontinuity modes are incorporated. On the other hand, the traction continuity condition across the discontinuity is enforced in the statically optimal form as the fine scale statics. The admissible discontinuity modes satisfying *a priori* the traction continuity condition are derived. In addition to the relative rigid body motions (translations and rotations), more general relative stretching modes that induce discontinuous strain/stress fields in the bulk, are obtained with no limit of a vanishing Poisson's ratio. The proposed model then particularizes to 2D quadrilateral elements. The displacement jumps at the discontinuity nodes are selected as the enrichment parameters and regarded as global variables shared by neighboring elements. The resulting model not only is intrinsically stress locking free for a fully softened discontinuity, but also automatically guarantees global continuity of the displacement jumps. Another benefit is that the numerical instability suffered under the unfavorable element/discontinuity configuration can be easily circumvented. Furthermore, the vanishing displacement jumps at the discontinuity tip can be enforced trivially as conventional essential boundary conditions and those complex strategies introduced in existing models are avoided. Together with the failure criterion related to the element average stress and the propagation orientation based on a simplified nonlocal stress, a local crack tracking algorithm is adopted to guarantee global continuity of the discontinuity path. Representative numerical simulations of element and structure

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benchmark tests are presented to verify the performance of the proposed approach in the modeling of arbitrary discontinuity propagation in solids.

Keywords: Computational failure mechanics; Finite element method; Strong discontinuities; Localized failure; Embedded cracks

1. Introduction

Softening responses of inelastic solids are accompanied with strain localization, i.e., a manifestation of concentration of defects such as cracks in concrete, joints in rocks, shear bands in soils, dislocations and slip lines in metals, etc. Structural collapse is often the consequence of such localized failure, characterized by the formation of localization bands of small or even fracture surfaces of negligible width compared to the length scale of the structure considered. Therefore, it is of utmost significance to evaluate (residual) structural safety once strain localization occurs, and to prevent potential catastrophic collapse caused by localized failure. To the above ends, the computational failure mechanics is nowadays playing an increasingly important role in the design and analysis of engineering structures.

Ever since the pioneering work of Ngo and Scordelis [1] and Rashid [2], different numerical approaches for the modeling of localized failure in solids have been proposed and some of them are well documented. For instance, on one hand, it is known that the smeared crack model is suffered from the problems of mesh-alignment dependence and spurious stress locking (i.e. non-vanishing stress transfer across a fully softened discontinuity) [3,4]. On the other hand, the inter-element discrete crack model [5,6] exhibits similar difficulties, and complex remeshing techniques in general have to be introduced. As alternatives, the nonlocal and gradient-enhanced models [7–9] are effective, if and only if rather fine spatial discretization is employed to resolve the incorporated small length scale.

During the last two decades, several advanced approaches have been proposed in the context of finite element method (FEM) to solve the aforementioned problems. Among them, the embedded finite element method (EFEM) with element enrichments [10–14] and the extended finite element method (XFEM) with node enrichments [15–19], have received a great mount of attention. Both EFEM and XFEM allow arbitrary propagation and resolution of discontinuities independently of the underlying spatial discretization. However, they are of completely different theoretical fundamentals and in general regarded as two irrelevant approaches; see Borja [20], Dias-da-Costa et al. [21], Jirásek and Belytschko [22], Oliver et al. [23]. In order to compare objectively both methods within a common framework, recently Wu [24] presented a unified analysis of enriched finite elements for cohesive cracks and applied it to Timoshenko/Euler–Bernoulli beams with softening hinges [25]. The unified kinematics of strong discontinuities is established based on the variational multiscale method [26,27] and the bridging scale method [28,29]. Namely, the displacement field around the strong discontinuity is hierarchically decomposed into two parts, i.e., a coarse scale one which is continuous over the whole domain and a fine scale one that exhibits displacement jumps across the discontinuity. A bridging scale containing information from both scales is introduced to make possible such a hierarchical decomposition. Dependent on the enriched discontinuity modes and the resulting relative displacement field, the kinematics employed in different models can be identified. Correspondingly, the statics (i.e. the equilibrium equations) is split into two coupled problems, i.e., a coarse scale one that can be numerically resolved by the standard FEM and a fine scale one in which the strong discontinuity is implicitly or explicitly accounted for. The treatment of fine scale kinematics and statics constitutes the most fundamental difference between variable versions of EFEM and XFEM.

The EFEM is closely related to the concept of assumed enhanced strain (AES) [30] upon which the standard finite element is enriched with elegantly constructed discontinuity modes. In earlier EFEMs [10–13,31–34], only the constant discontinuity modes, caused by relative rigid body translations, were accounted for. This is not a problem for the underlying constant strain triangular or tetrahedral (CST) element. However, for high-order elements like quadrilateral bilinear Q1, the constant discontinuity modes are insufficient and spurious stress locking occurs, particularly, in dominantly bending cases [35]. To improve the numerical performance, linear discontinuity modes have been incorporated in the kinematics [36–41]. Another peculiarity of the EFEM is that, the enrichment parameters, usually regarded as element-wise local variables, are eliminated at the element level by static condensation. This procedure yields a global stiffness matrix with the same bandwidth as that in the standard FEM, but global continuity of displacement jumps along the discontinuity path cannot be enforced easily. Contrariwise, the XFEM employs the local or global partition