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A dual Lagrange method for contact problems with regularized frictional contact conditions: Modelling micro slip

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Abstract

This paper presents an algorithm for solving quasi-static, non-linear elasticity contact problems with friction in the context of rough surfaces. Here, we want to model the transition from sticking to slipping also called micro slip in a physically correct way in order to reproduce measured frictional damping. The popular dual Mortar method is used to enforce the contact constraints in a variationally consistent way without increasing the algebraic system size. The algorithm is deduced from a perturbed Lagrange formulation and combined with a serial–parallel Iwan model. This leads to a regularized saddle point problem, for which a non-linear complementary function and thus a semi-smooth Newton method can be derived. Numerical examples demonstrate the applicability to industrial problems and show good agreement to experimentally obtained results.

Keywords: Contact with friction; Dual Mortar methods; Micro slip; Rough surfaces; Constitutive contact equations; Iwan model

1. Introduction

Since solving non-linear contact problems within the FEM framework is still a challenging task from both the mathematical and the engineering point of view, a lot of research has been done in this field in the past decades. For an overview to contact problems in general, we refer to the monographs by Johnson [1] or Bowden/Tabor [2] for contact between rough surfaces, the monographs by Kikuchi/Oden [3] and Eck [4] for existence and uniqueness results and to the monographs by Willner [5], Laursen [6] and Wriggers [7] for computational aspects.

The most popular approaches to enforce the contact condition are the penalty method and the Lagrange multiplier method. Within the penalty framework, a spring between a slave location and a master surface is generated modelling the contact traction $t_c^s = G_n(u) = a_n \cdot u_n$ as a function of the displacements with the spring constant a_n, if the non-penetration condition is violated. This approach is often combined with a node-to-surface formulation. Though easy to implement, since the contact forces do not enter as an extra variable in the system, it has the drawback of an

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ill-conditioned matrix system for stiff contact laws, i.e., $a_n \to \infty$ and does not satisfy the patch test. The Lagrange multiplier method introduces the contact traction as an extra variable with the benefit of being able to model arbitrary stiff contact laws. Combined with a surface-to-surface discretization, the Lagrange multiplier formulation leads to more robust algorithms, since the contact condition is not enforced point-wise in the strong form, but weakly in an integral formulation over the surrounding surface of a point. The drawback, however, is the need for solving a saddle point system with respect to the displacements and the contact traction in every Newton–Raphson iteration.

If contact problems between two deformable bodies and non-matching meshes are considered, the Lagrange multiplier setting is strongly related to mortar methods introduced originally in the context of domain decomposition techniques for non-matching meshes by Bernardi [8] and Belgacem [9]. The Mortar methods are also characterized by the introduction of an additional variable to model the traction across the interface and a weak formulation of the boundary constraints leading to a surface-to-surface approach. It was adapted to small deformation contact problems by Belgacem [10]. The idea of the weak formulation of the contact condition was later expanded to large deformation contact [11,12], for curved interfaces and large sliding [13–15] and dynamical problems [16,17].

Among the Mortar methods, the dual Mortar method [18,19] has become of great interest for solving contact problems in recent years [20–23] since it enforces the non-penetration condition in a variationally consistent way without increasing the system size. Due to a smart choice of the basis functions for the contact traction, the Lagrange multiplier can be condensed from the global system before solving without losing the optimality of the solution. Combined with a semi-smooth Newton method [24,25], which interprets the contact conditions as a semi-smooth non-linear function, one obtains a powerful tool for contact problems since all non-linearities of the system (material and contact conditions) can be handled within the same iteration loop.

So far a perfect Coulomb law has been considered in most publications using the Mortar formulation. More complicated friction models like thermomechanically coupled friction were explored by Hüber [26] and Temizer [27]. Yang and Laursen [28] have considered the full Reynolds approximation for lubricated contact. But for use in structural dynamics it is important to model the transition between sticking and slipping, also called micro slip, in a physical correct way [29,30]. With this smooth friction model one can reproduce measured frictional damping [31], like for example in the bolted connection in the casing of a jet turbine. Thus there is a need for more general contact laws, such as constitutive contact laws for rough surfaces, in combination with suitable numerical algorithms.

Traditionally constitutive contact laws are realized within a penalty framework with all resulting drawbacks mentioned above. Thus we propose a new generalized dual Mortar formulation to solve numerically problems with regularized frictional contact conditions. In the Lagrange multiplier framework, the smooth transition between sticking and slipping can be easily realized and due to the dual formulation the Lagrange multipliers can be condensed from the global system. Therefore, we extend the ideas presented in our previous paper [32] to regularized frictional contact laws.

The outline of the present work is as follows: In Section 2, the fundamental equations in the strong form and their weak formulation are summarized. The contact behaviour of rough surfaces and their realization within the dual Mortar framework is presented in Section 3. In Section 3.1 an elastic stick part is added to the Coulomb friction law and a semi-smooth Newton for the active set iteration is derived. Finally, the Lagrange multipliers are locally eliminated from the matrix representation and the displacement based reduced algebraic system is given. In Section 3.2 the ideas from Section 3.1 are combined with a serial–parallel Iwan model [33] in order to get a smooth transition from sticking to slipping. Again a semi-smooth Newton is derived. Section 4 illustrates the robustness and applicability to real life contact problems of the newly derived dual Mortar formulation. First, we consider a simple mathematical benchmark for which analytical solutions exist, and then we present a resonator example with a bolted connection, for which measurements exist. Some conclusions are given in Section 5.

2. Problem definition

2.1. Strong form

We start with a brief overview of the $3D$ frictionless two-body contact problem in a non-linear elasticity setting [3,5–7]. As one can see in Fig. 1, two contacting bodies are considered, a slave body, denoted by s, and a master body, denoted by m, with domains $\Omega^\alpha \in \mathbb{R}^3$, $\alpha = s, m$ in the reference configuration. The bodies are undergoing motion