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Transient response analysis of randomly parametrized finite element systems based on approximate balanced reduction

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Highlights

- We propose a model reduction technique for large scale stochastic finite element systems.
- The reduced basis spans the dominant eigenspace of the stochastic controllability Gramian.
- Computationally efficient iterative Arnoldi–Lyapunov basis building methods for large stochastic systems.
- Implicit restart scheme for Arnoldi–Lyapunov vector basis has been proposed.
- Transient response analysis of large dynamical systems illustrated with numerical examples.

Abstract

A model order reduction scheme of the transient response of large-scale randomly parametrized linear finite element system in state space form has been proposed. The reduced order model realization is aimed at preserving the invariant properties of the dynamic system model based on the dominant coupling characteristics of the specified system inputs and outputs. An *a-priori* model reduction strategy based on the balanced truncation method has been proposed in conjunction with the stochastic spectral Galerkin finite element method. Approximation of the dominant modes of the controllability Gramian has been performed with iterative Arnoldi scheme applied to Lyapunov equations. The reduced order representation of the randomly parametrized dynamical system has been obtained with Arnoldi–Lyapunov vector basis using an implicit time stepping algorithm. The performance and the computational efficacy of the proposed scheme has been illustrated with examples of randomly parametrized advection–diffusion–reaction problem under the action of transient external forcing functions. The convergence of the proposed reduced order scheme has been shown with a-posteriori error estimates.

Keywords: Stochastic transient response; Balanced truncation; Stochastic spectral Galerkin; Controllability Gramian

1. Introduction

Uncertainty in the mathematical modeling of engineering systems has been an active area of research over the past few decades which focuses on the statistical quantification of the effect of input uncertainty on the response quantities

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of interest. This resolution of the stochastic mathematical models and the propagation of uncertainty has called for efficient numerical methods to tackle these expensive problems which range from non-intrusive efficient Monte-Carlo and quasi-Monte Carlo techniques to the intrusive stochastic spectral Galerkin methods. Excellent review articles have summed up in the research in this domain [1,2]. It should be noted though that the above stochastic systems are different from the classical "stochastic differential equations". In the latter case, the random inputs are in the form of idealized processes (such as Wiener process, Poisson process, to name a few) and the stochastic calculus used for their study is a mature subject of active research [3]. In the present work we have considered the uncertain inputs to be in the form of random parameters associated with the system of governing partial differential equations describing the physical system to be studied.

Uncertainty in coupled multiphysics linear time-invariant (LTI) systems have been found to have a huge impact on their control performance [4]. The present work focuses on the resolution of the randomly parametrized LTI systems using efficient reduced order modeling techniques. Thus the first part of the article gives the key concepts employed in the description and quantification of the random parameters in stochastic partial differential equations (SPDE) and the incorporation of this description within the stochastic finite element setup. The solution techniques employed for the stochastic linear system can be broadly classified into two broad categories: (a) the non-intrusive stochastic sample based simulation techniques and (b) intrusive stochastic spectral Galerkin methods.

Various Monte Carlo Simulation (MCS) techniques belong to the class of non-intrusive methods and have been widely used. These methods have the virtue of easy implementation and trivial parallelization but the convergence of the solution statistics is slow with the mean value converging as $1/N_s$ where N_s is the number of random realizations. Sometimes the convergence can be accelerated by improved sampling techniques (such as the importance sampling, Latin hypercube sampling, orthogonal sampling, to name a few) such as the "variance reduction techniques" [5] or the response surface method or the experiment design method. The limitations of these techniques are generally dictated by the dimension of the input stochastic space.

Alternatives to MCS methods can provide us with an explicit functional relationship between the independent input random variables and hence can allow easy evaluation of functional statistics or probabilities. Non-statistical approaches based on a perturbation method [6], the Neumann expansion method [7,8] estimates the response surface in a parameter space. On the other hand the Galerkin-type methods [9–12] developed with differing choice of the approximation space, systematically lead to a high precision solution allowing the response to be expressed explicitly in terms of the basic random variables describing the uncertainties. Their principal drawback lies in the fact that the dimensionality of the resulting system of linear equations is huge. The difficulty to build efficient preconditioners and memory requirements induced by these techniques are still challenging and active areas of research.

The additional computational overhead associated with obtaining the response statistics of the randomly parametrized systems have motivated researchers to look into various model reduction techniques for the numerical solution of SPDE. A review of some of these techniques can be found in [13,14]. Some of these techniques attempt to perform a spectral (Hilbert Karhunen–Loève) decomposition of the stochastic solution to obtain the set of basis functions [15] or use a low-order Neumann expansion scheme to compute an estimation of the correlation structure of the response vector [11]. These belong to the class of *a-posteriori* model reduction since the optimal basis is calculated from a primary approximation of the statistics of the stochastic response. On the other hand the *a-priori* model reduction schemes in the context of Galerkin spectral stochastic methods evaluate the stochastic basis functions for approximating the solution using well defined optimality criterion. Methods belonging to this category are Generalized Spectral Decomposition [16] and the so called Reduced Basis methods [17].

On the other hand, the problem of reduced order modeling for linear time invariant systems (LTI) has been studied widely within the scope of control literature [18,19]. The foundation for the minimal realization of LTI systems using balanced truncation has been laid in [20] which is a principal components analysis of the LTI system using the concept of observability and controllability Gramians. Among the vast range of other model reduction techniques for LTI systems we refer to the singular value decomposition based approaches [21], the classical moment matching techniques [22] and singular perturbation technique [23] for the attention they have received. Model reduction for systems with random inputs modeled as stochastic processes have been studied in [24,25].

The objective of this study is to approach model reduction from a systems perspective where the complete information of the LTI system is available in the form of a finite element model obtained from applying the stochastic spectral Galerkin method to a randomly parametrized stochastic partial differential equation. These systems typically have very large dimension and it is a challenge to realize their transient response statistics with an appropriate