Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 285 (2015) 694–713
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

A mesh regularization scheme to update internal control points for isogeometric shape design optimization

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Received 10 September 2014; received in revised form 7 November 2014; accepted 28 November 2014
Available online 9 December 2014

Highlights

- Efficient mesh regularization scheme that avoids mesh entanglement.
- Bijective mapping due to the convexity of Dirichlet energy functional.
- High quality of domain parameterization in isogeometric shape design optimization.
- Uniformity achieved by minimizing the variance of Jacobian.
- Orthogonality using a dimensionless measure instead of stretching energy functional.

Abstract

This paper presents a variational method to update internal control points in isogeometric shape optimization. The important properties of domain parameterization such as bijective mapping between parametric and physical domains, uniform mesh, and orthogonal mesh are enforced simultaneously. The bijective mapping is achieved by minimizing a Dirichlet energy functional. To prevent the divergent behavior of the minimizing process due to the severely distorted initial mesh, a constraint is introduced to enforce the positive Jacobian of mapping from parametric to physical domains. In spite of adding the constraint that might increase computational costs, the proposed method is more efficient due to the convexity of Dirichlet energy functional, compared with the other unconstrained methods. Also, it turns out that the proposed method is more effective to achieve the bijective mapping, especially near a concave boundary. The uniform parameterization of the domain is achieved by minimizing the Dirichlet energy functional and the orthogonality of mesh is performed by minimizing a dimensionless functional. The required design sensitivity of the employed functional and constraint is derived with respect to the position of internal control points. The developed scheme of mesh regularization is effective to maintain the high quality of domain parameterization during the shape optimization process.

Keywords: Isogeometric shape optimization; Internal control point; Dirichlet energy functional; Bijective mapping; Uniformity; Orthogonal mesh

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http://dx.doi.org/10.1016/j.cma.2014.11.045
0045-7825/

1. Introduction

To resolve the discrepancy between the finite element model and the computer aided design (CAD) model, Hughes et al. [1] developed the isogeometric analysis (IGA) method, which is an analysis framework employing the same basis function as used in the CAD system. Instead of the mesh quality in the conventional finite element analysis (FEA), an analogous concept of model quality exists within the IGA (Cohen et al. [2]). The model quality indicates the parameterization quality of the physical domain, which has been shown to play a significant role in the accuracy of the isogeometric analysis as mesh quality does in the conventional finite element analysis. Lipton et al. [3] investigated the effect of degenerating control net on the accuracy of IGA. Also, Xu et al. [4] showed that the quality of domain parameterization significantly affects the solution accuracy, based on the IGA of heat conduction problems. One of the major causes of the invalid parameterization of domain, which indicates negative Jacobian of mapping from parametric to physical domain exists in the domain, is the large variation of the physical domain during the simulation process. *Especially in the shape design optimization procedure, it is still an inherent challenge to update domain parameterization after boundary variation, in order to avoid mesh distortion and maintain high quality of domain parameterization for reliable response and design sensitivity analysis*.

In the shape design optimization based on the conventional finite element analysis, re-meshing is unavoidable if the boundary variation is significantly large and complicated. However, re-meshing should not be frequently used, since the addition of finite elements may lead to the sudden variation of the objective function or violation of the constraints, which prevents smooth convergence to an optimal shape. To minimize the use of remeshing, Belegundu and Rajan [5] choose the magnitude of a set of fictitious loads applied on the structure as design variables, and the nodal displacements obtained by solving the linear elasticity problem under these fictitious loads are added to the initial mesh to update the design. Yao and Choi [6] consider the design variation on the boundary as prescribed displacement, and find the nodal displacements of internal nodes by solving the linear elasticity problems to update the internal mesh. However, Hsu and Chang [7] show that using homogeneous material properties in the aforementioned methods utilizing the solution of linear elasticity problem may result in severe mesh distortion due to over-stiffening or under-stiffening of some elements. To overcome this problem, they proposed a method to perform two consecutive linear elastic finite element analyses. The first analysis is performed with the homogeneous elastic property, and the second analysis is performed based on the non-uniform elastic property determined, using a fully stressed design method, from the result of the first analysis. However, in boundaries with large variations, it is fundamentally unavoidable to incrementally increase the boundary variation; and for each increment, two linear elastic analyses are required to be performed, which may significantly increase the computational cost of the shape design optimization.

In the isogeometric analysis framework, there have been several researches to construct high quality domain parameterization, based on the boundary geometry obtained from CAD system. Xu et al. [4] suggest a mesh regularization scheme using constrained optimization to distribute the internal control points, considering the uniformity, orthogonality and injectivity of the domain parameterization. The uniformity and orthogonality are realized through minimizing the bending and stretching energy functionals, respectively. Also, the injectivity is realized through imposing the positivity of the Jacobian control scalars. Since it requires large computational costs to evaluate the Jacobian control scalars, they utilize a method that approximately tests the sign of the Jacobian control scalars based on conic-hull hodograph. The conic-hull hodograph based method is efficient, but is not usable if the cone of the given boundary is non-transverse due to its complicated geometry. For detailed explanations of the methods to prevent self-intersection of the CAD object using the Jacobian control scalar and conic-hull hodograph, interested readers may refer to Gain and Dodgson [8]. Also, this mesh regularization scheme using constrained optimization is applied to three dimensional solid model in Xu et al. [9,10]. Wang and Qian [11] maximize the minimal Jacobian control scalar to obtain valid domain parameterization of a trivariate B-spline solid through constrained optimization method. They also utilize the constraint aggregation strategy to reduce the number of constraints. Instead of using constraints on the Jacobian control scalar to impose the positivity of the Jacobian in the whole domain, Xu et al. [12] suggest the unconstrained optimization method, based on the harmonic mapping theory. They derive a functional from the Laplace's equation, under the assumption that Jacobian is non-vanishing in the domain. However, a stationary solution of minimizing this functional is not guaranteed to satisfy the Laplace's equation, and gives a significantly deteriorated domain parameterization, especially near the concave boundary. *This drawback is investigated in this paper, through comparison with the results, obtained from minimizing the Dirichlet energy functional*. Furthermore, they obtain the sensitivity of the suggested functionals with respect to the position of the internal control point by using the finite difference method,