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Mixed stabilized finite element methods in nonlinear solid mechanics. Part III: Compressible and incompressible plasticity

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Abstract

This paper presents the application of a stabilized mixed strain/displacement finite element formulation for the solution of nonlinear solid mechanics problems involving compressible and incompressible plasticity. The variational multiscale stabilization introduced allows the use of equal order interpolations in a consistent way. Such formulation presents two advantages when compared to the standard, displacement based, irreducible formulation: (a) it provides enhanced rate of convergence for the strain (and stress) field and (b) it is able to deal with incompressible situations. The first advantage also applies to the comparison with the mixed pressure/displacement formulation. The paper investigates the effect of the improved strain and stress fields in problems involving strain softening and localization leading to failure, using low order finite elements with continuous strain and displacement fields ($P1P1$ triangles or tetrahedra and $Q1Q1$ quadrilaterals, hexahedra, and triangular prisms) in conjunction with an associative frictional Drucker–Prager plastic model. The performance of the strain/displacement formulation under compressible and nearly incompressible deformation patterns is assessed and compared to a previously proposed pressure/displacement formulation. Benchmark numerical examples show the capacity of the mixed formulation to predict correctly failure mechanisms with localized patterns of strain, virtually free from any dependence of the mesh directional bias. No auxiliary crack tracking technique is necessary.

Keywords: Mixed finite elements; Stabilization; Plasticity; Strain softening; Strain localization; Mesh dependence

1. Introduction

In previous works [1,2], the authors have formulated stable mixed stress/displacement and strain/displacement finite elements with equal order interpolation for the solution of nonlinear problems in solid mechanics. The proposed formulation uses the sub-grid scale approach to circumvent the restrictiveness of the inf–sup compatibility conditions on the choice of the interpolation spaces. The objective of such formulation is to achieve a discrete scheme with enhanced stress accuracy. This means that the mixed formulation displays a global rate of convergence on stresses higher

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than the corresponding irreducible formulation. Such improvement of the convergence estimates also applies at local level. And this characteristic proves to be crucial in strain localization problems involving softening materials.

Strain localization inevitably occurs in softening materials subjected to monotonic straining. Once the peak stress is reached, and upon continuing straining, the stress decreases and strains concentrate inside a narrow band of material while the material outside the band unloads elastically. As the localization progresses, the width of the localization band diminishes and, unless there is a microstructural limitation, it tends to zero. The particular components of the strain tensor that localize during this process depend on the specific constitutive behavior of the material. In Rankine-type materials, only normal elongations localize, eventually forming tensile cracks; if the nonlinear behavior is incompressible, shear strains concentrate, leading to slip surfaces.

Quasi-singular strain or stress states occur at the vicinity of the propagating cracks or slip lines. For linear elements and even in elastic behavior, it is well known that the standard irreducible formulation fails to provide guarantee of local convergence of stress values in such situations, such as the tip of a notch or a propagating crack. And this lack of local convergence leads to the spurious mesh bias dependence often displayed by standard finite elements when using local softening constitutive models. Contrariwise, the proposed mixed formulations do provide the necessary guarantee of convergence for local stress convergence. This characteristic proves to be sufficient to avoid mesh bias dependence of the numerically computed failure mechanisms and responses.

In Ref. [2,3], the mixed strain/displacement formulation was applied in conjunction with an isotropic Rankine damage model, formulated in secant form, to model problems of tensile cracking propagation and failure. It was observed there that: (a) the resulting discrete FE model is well posed and stable, (b) the formulation is convergent and, on mesh refinement, it approaches the original continuum problem, and (c) the results obtained are not spuriously dependent of the finite element mesh used; they depend only on the actual material model (damage criterion in this case) adopted. This represented a significant advancement in the solution of such problems, particularly considering two noteworthy features of the approach. On one hand, it is of general application, in 2D and 3D problems, to structured and unstructured meshes and to simplicial or non simplicial elements. On the other hand, no "ad hoc" auxiliary crack tracking technique is necessary. However, the application of the proposed formulation to problems involving local softening plasticity models remained open.

In previous works, the authors have applied stabilized mixed displacement–pressure methods [4–9] to the solution of J_2 elasto-plastic problems with simplicial elements. In J_2 dependent problems, the plastic flow is isochoric and the main challenge for the discrete formulation is the incompressibility constraint. Unless this is properly dealt with, spurious pressure oscillations appear and the discrete solution is totally polluted. A stabilized mixed formulation provides a discrete problem which is fully stable, even for problems involving localization of shear strains and the formation of slip lines. The results obtained, both in terms of collapse mechanism and global load–deflection response, compare very favorably with those obtained with the standard irreducible formulation, which almost inevitably shows an unacceptable mesh dependence. Nevertheless, regarding the computation of the deviatoric stresses, the stabilized mixed pressure–displacement formulation has the same convergence behavior than the irreducible formulation. This is because, in both formulations, the discrete deviatoric strains are computed by direct differentiation of the discrete displacement field. This means that the corresponding convergence rate is necessarily one order less than that of the displacements. When using linear interpolation for the displacements and in quasi-singular situations, this may prove to be insufficient. The remedy is to use an independent interpolation, linear at least, not only for the volumetric part of the strain (or stress) tensor, but for all of its components.

Therefore, the objectives of this paper are five: (1) to extend the stabilized mixed strain/displacement formulation to plasticity problems, (2) to investigate the effect of the improved strain and stress fields in problems involving strain softening and localization leading to failure, (3) to assess the performance of the formulation under nearly incompressible deformation patterns, (4) to compare the performance of the proposed formulation with the previously proposed pressure/displacement formulation and (5) to show that the formulation is applicable in 2D and 3D, to structured or unstructured meshes of triangles, quadrilaterals, tetrahedra, hexahedra or prisms. Both pressure sensitive and incompressible plasticity models are contemplated. To achieve this, the Drucker–Prager plasticity model is selected as target model, as it may incorporate pressure sensitivity through the friction angle of the material, as well as reduce to a pure cohesional behavior when null friction is assumed.

Inelastic plastic flow is a directional phenomenon. In the stress space, assuming associative plasticity, it occurs in the direction normal to the yield surface; in non-associative plasticity, the directionality of the flow is established from a plastic potential, different from the yield criterion. In any case, plasticity does not occur isotropically. This