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A solution to transient seepage in unsaturated porous media

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Abstract

This paper presents a solution to seepage problems in porous media considering the complete time-dependent transition from fully saturation to partially unsaturated states and *vice-versa*; therefore capturing the evolution of the free surface (region with zero liquid pressure). A simple and efficient method to implement the seepage face boundary condition for finite element solutions is proposed. The method is based on an analogy to unilateral constraints in Plasticity and, in essence, adds some extra unknowns to the finite elements with boundaries near the seepage face. The free surface is thus automatically predicted. The resulting enriched elements can also account for ponding or infiltration at the external surface. The solution is accomplished by considering the theory of porous media with slightly compressible liquids. The formulation can easily accommodate liquid retention models with hysteresis. Verification examples are presented in addition to simulations of drainage and infiltration illustrating the capabilities of the proposed solution.

Keywords: Transient analysis; Unilateral boundary conditions; Free surface; Infiltration; Ponding; Unsaturated soils

1. Introduction

Seepage problems are an important topic in engineering and have particular applications to groundwater simulations or geotechnical engineering. For instance, it is common practice to import data from seepage analyses into stability codes for risk assessment of slopes and retaining walls. Analytical solutions are available for simple situations [1,2] and geometric configurations and numerical methods are usually employed to the more complex cases.

Among the numerical methods for seepage analyses, the finite element is perhaps the mostly used in the industry and by researchers [3]. One major difficulty for analytical or numerical solutions is the representation of the unilateral constraints at the seepage face. To circumvent this problem, a number of strategies has been proposed in the literature and some include iterative techniques while others use mesh adaptivity [4–7]. The topic is still active with some recent papers published [8–12].

The modelling of fluid flow in unsaturated soils can be consistently derived from the theory of porous media (TPM) [13]. The TPM is a continuum mechanics approach in which a macroscopic formulation is obtained for each constituent in addition to the whole mixture. This theory has its origins on the concept of volume fractions and the

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theory of mixtures and has been improved along the years resulting in a rational methodology to represent the complex behaviour of porous media in a simple way [14].

One important aspect to be considered when modelling unsaturated porous media is the so-called water retention curve (or liquid retention model—LRM) [15–21]. This curve represents the link between liquid saturation and capillary pressure hence closing the system of equations in addition to introducing each particular material behaviour into the computational model. One common observed characteristics of the LRM is the hysteresis and scanning curves that appear during experiments with cycles of drainage and imbibition. These may cause difficulties in the numerical analyses; however the presented formulation can easily accommodate hysteretic models [22].

This paper is organised as follows. In Section 2, a brief review of the theory of porous media is presented with focus on deriving the governing equation for seepage analyses. In Section 3, the treatment of unilateral constraints for the seepage face and corresponding solution with enriched elements are proposed. In Section 4, constitutive models for the liquid retention behaviour and the liquid conductivity are introduced. In Section 5, some verification simulations are discussed and the predictive capabilities are illustrated. Finally, the conclusions are given in Section 6.

In this paper, the order of a tensor variable is indicated by the number of dots added under the corresponding symbol—*underdot* as diacritic marks in phonetics. For instance, $\underset{\cdot}{\boldsymbol{a}}$ and $\underset{\cdot\cdot}{\boldsymbol{a}}$ are (different) tensors of first and second orders, respectively—confusion is avoided because of the underdots. Inner products are expressed with operator dots such as in $s = \underset{\cdot}{\boldsymbol{a}} \cdot \underset{\cdot}{\boldsymbol{a}}$ and $\sigma = \underset{\cdot\cdot}{\boldsymbol{a}} : \underset{\cdot\cdot}{\boldsymbol{a}}$. The dyadic product is expressed as $\underset{\cdot\cdot}{\boldsymbol{a}} = \underset{\cdot}{\boldsymbol{a}} \otimes \underset{\cdot}{\boldsymbol{a}}$.

2. Governing equations and numerical solution

The governing equations are derived with basis on the theory of porous media (TPM). The solution is then obtained with the finite element method [3,23,24]. Further details, especially on the derivation of all equations for porous media, are found in the works by de Boer, Ehlers, Lewis, Schrefler and Pedroso [13,14,25,22].

2.1. Governing equations

In the TPM, each material constituent $\alpha := \{s, \ell, g\}$ (solid, liquid, gas) in a representative elementary volume (REV) is characterised by its volume fraction n_α which is then used to define a partial density quantity ρ_α by means of weighting the real density ρ^α. For the fluid constituents $\beta := \{\ell, g\}$, it is also convenient to define the saturation s_β and pressure p_β. Another useful quantity is the volume fraction of fluids n_f which is equal to the porosity when all pores are interconnected. These key quantities are summarised in Eq. (1)

$$n_\alpha = \frac{dv_\alpha}{dv} \qquad \rho_\alpha = n_\alpha \, \rho^\alpha \qquad s_\beta = \frac{dv_\beta}{dv_v} \qquad n_f = \frac{dv_v}{dv} \tag{1}$$

where dv is the volume of the REV, dv_α the volume of each particular constituent, and dv_v the volume of all fluid constituents in the REV; for instance, with liquids and gases: $dv_v = dv_\ell + dv_g$. We also define the partial density of the mixture ρ by adding the three partial densities. For a three-constituents system, ρ and other easily verifiable expressions can be written as follows

$$\rho = \rho_\ell + \rho_s + \rho_g \qquad n_f = n_\ell + n_g \qquad n_\ell + n_g + n_s = 1 \qquad \rho_\beta = n_f \, s_\beta \, \rho^\beta. \tag{2}$$

An important concept required to close the system of equations is the liquid retention model. With the following definition of capillary pressure

$$p_c = p_g - p_\ell \tag{3}$$

the liquid retention model can be expressed by

$$C_c = \frac{\mathrm{d}s_\ell}{\mathrm{d}p_c}. \tag{4}$$

For the following derivations—and the whole extent of this paper, the gas phase is disregarded with the pressure on the gas being assumed equal to zero; thus $p_g = 0$, $\dot{p}_g = 0$ and $p_c = -p_\ell$.

The flow of fluids within the porous medium is best expressed in terms of an Eulerian framework in which the solid skeleton serves as reference. For the seepage modelling, the solid matrix is considered to be perfectly rigid and the seepage velocity of liquid $\underset{\cdot}{\boldsymbol{w}}_\ell$ is introduced. The relative velocity $\underset{\cdot}{\boldsymbol{w}}_\ell$ allows for the definition of a viscous interaction