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Algebraic distance estimations for enriched isogeometric analysis

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Abstract

In problems with evolving boundaries, interfaces or cracks, blending functions are used to enrich the underlying domain with the known behavior on the enriching entity. The blending functions are typically dependent on the distance from the propagating boundaries. For boundaries defined by free form curves or surfaces, the distance fields have to be constructed numerically. This may require either a polytope approximation to the boundary and/or an iterative solution to determine the exact distance to the boundary. In this paper a purely algebraic, and computationally efficient technique is described for constructing distance measures from Non-Uniform Rational B-Splines (NURBS) boundaries that retain the geometric exactness of the boundaries while eliminating the need for iterative and non-robust distance calculation. The constructed distance measures are level sets of the implicitized constituent Bezier patches of the NURBS surfaces that are obtained purely algebraically. Since, in general, the implicitized functions extend beyond the parametric range of the generating Bezier patch, algorithmic procedures are developed to trim these global implicit functions to the boundaries of the Bezier patch. Boolean compositions are then carried out between adjoining Bezier patches to construct a composite distance field over the domain. The compositions rely on R-functions that are also algebraic in nature. The developed technique is demonstrated by constructing algebraic distance field for complex geometries and by solving a variety of examples culminating in the analysis of steady state heat conduction in a solid with arbitrary shaped three-dimensional cracks.

Keywords: Algebraic geometry; NURBS; Implicitization; R-functions; Distance field; Enrichments

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1. Introduction

In general, in moving boundary problems, the motion of complex boundaries need to be tracked within the domain. Examples of such problems arise in many fields, including fluid mechanics, solid mechanics, optimal design, computer vision and image processing. Commonly, an Eulerian framework in which the geometry of the boundaries is inferred implicitly as the zero level set of an evolving field is used to numerically solve these class of problems [1]. Since the level sets implicitize both the geometry of the boundary as well as the distance from the boundary, in other computational procedures for moving boundary problems, there is a common need for explicitly calculated distance fields.

In general, in these problems, distance from the boundary or interface serves as a measure of influence of the behavior on the boundary at a point in the underlying domain. Computational procedures relying on distance fields are

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many. For example, the use of distance fields for image processing is well established [2]. In fluid–structure interaction problems, signed distance functions have been used to represent fluid boundaries such as the fluid–structure interface or the free surface within the computational domain [3,4]. Distance fields are also useful in contact problems [5–7] for defining the gap function for contact detection. Recently, in the Isogeometric Analysis (IGA, [8,9]) literature, an approximate distance field has been used to enrich the base approximations with those on lower-dimensional geometrical features, enabling application of boundary conditions as well as simulations of crack propagation [10]. Mathematical representation of graded materials have also used distance fields to describe the desired blended material distributions [11,12]. Thus, while the use of distance field as a measure of influence of the boundaries on the domain has enabled the numerical solution to complex problems, inexpensive distance field calculations are essential for the analysis to be computationally efficient.

Since the notion of distance fields is fundamental to many computational solution procedures, there exists a significant established literature aimed at improving the techniques for calculating the distance field. Iterative methods such as the Newton–Raphson technique are usually required for finding continuously varying distance from spatial points to a parametric surface [13–16]. While the iterative schemes make distance computations expensive, the calculated distance field is unfortunately not sufficiently smooth for many engineering applications. This is since the distance function is not differentiable at points that are equidistant to two or more points on the surface (cut locus of the surface).

Iterative distance calculation methods in general become necessary due to the geometrical nonlinearity of the surface. Hence, piecewise planar approximations of a parametric surface using, for instance, triangular mesh are popular in computer graphics applications (see for instance [17]). The piecewise linearization of geometry makes the distance calculation relatively straightforward, albeit by introducing a new combinatorial problem — that of determining the nearest planar surface from among the many possible ones at a spatial point. Therefore, the algorithms relating to piecewise planar approximations mainly deal with efficient data structures to determine the triangular face closest to the point of interest (see for instance, [18,19]) or algorithmically propagating distances from calculations closer to the surface [20,21]. The disadvantage of the piecewise planar approximations is that the geometrical exactness of the boundary is not preserved making the calculated distance accurate only in the limit of refinement of the planar approximations. But, more critically, the calculated distance field is non-smooth since a subset of the spatial points will project to an edge or vertex of the triangular face. This in turn implies that one cannot rely on distance calculations on planar approximations to generate smooth, continuous distance measures.

In general, distance measures may be thought of as extending approximations of fields from the boundaries into the underlying domain. For constructing such a distance measure, the exact distance to the boundary is not as critical as a smooth and monotonically increasing function of distance that serves as a measure of influence of the boundary on the underlying domain. Smoothness of such an approximate distance measure would ensure robustness of any calculations based on the constructed field. Hence, an approximate distance measure that is smooth is sufficient for these applications. Towards building such an approximate measure, Biswas and Shapiro [22] described a method using piecewise planar geometric approximations of a parametric curve. They constructed an approximate distance field for each linear segment and combined the distance field of each linear segment using R-functions into a global approximate distance field. The advantage of this technique over the previous distance calculation techniques is that this enables desired smoothness in the distance field. However, this technique relies on piecewise linearization of the geometry and hence, compromises the exactness of the boundary.

In this paper, a purely algebraic, and therefore computationally efficient, technique for constructing an approximate distance function that avoids the iterative, and therefore inefficient, distance computations is developed. The technique preserves the geometric exactness of low-degree (two or three) NURBS surfaces. Such purely algebraic distance computation procedures that preserve the geometric exactness do not appear to exist in prior literature. The proposed technique overcomes the need for the iterative exact distance computations at every quadrature point during analysis while providing smoother, more robust distance field relative to the numerically computed distance.

The rest of the paper is organized as follows. In Section 2, a general theoretical formulation for the algebraic distance field construction technique is presented and illustrated using rational Bezier as well as NURBS curves. In Section 3, a detailed algorithm is proposed for algebraic distance field construction technique for Bezier and NURBS curves. The properties of the algebraic distance field are discussed next in Section 4. In Section 5, the algebraic distance field algorithm is extended to three dimensional Bezier and NURBS surfaces. In Section 6, examples of algebraic distance field are demonstrated. The paper is finally summarized in Section 7.