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The virtual element method for discrete fracture network simulations☆

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Abstract

In this work, an optimization based approach presented in Berrone et al. (2013, 2014) [10–12] for Discrete Fracture Network simulations is coupled with the Virtual Element Method (VEM) for the space discretization of the underlying Darcy law. The great flexibility of the VEM in handling rather general polygonal elements allows, in a natural way, for an effective description of irregular solutions starting from an arbitrary triangulation, which is built independently of the mesh on other fractures. Only partial conformity is in fact obtained with this approach. Numerical results performed on several DFN configurations confirm the viability and efficiency of the resulting method.

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1. Introduction

Subsurface fluid flow has applications in a wide range of fields, including e.g. oil/gas recovery, gas storage, pollutant percolation, water resources monitoring, etc. Underground fluid flow is a complex heterogeneous multi-scale phenomenon that involves complicated geological configurations. Discrete Fracture Networks (DFNs) are complex sets of planar polygonal fractures used to model subsurface fluid flow in fractured (porous) rocks. Typically, a DFN is obtained stochastically using probabilistic data to determine a distribution of orientation, density, size, aspect ratio, aperture and hydrological properties of the fractures [1–3], and it is a viable alternative to conventional continuum models in sparse fracture networks. DFN simulations are very demanding from a computational point of view and due to the uncertainty of the statistical data, a great number of numerical simulations are required. Furthermore, the resolution of each configuration requires vast computational effort, increasing greatly with problem size. In this work,

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we focus on the resolution of the steady-state flow in large fracture networks. The quantity of interest is the hydraulic head in the whole network, which is the sum of pressure and elevation and is evaluated by means of the Darcy law. We consider the rock matrix as impervious and fluid can only flow through fractures and traces (intersections of fractures), but no longitudinal flow along the traces is allowed. Matching conditions need to be added in order to preserve continuity along traces and flux balance at fracture intersections. The classical approach to DFN simulations consists in a finite element discretization of the network and in the resolution of the resulting algebraic linear system. With this approach, a great numerical obstacle to overcome is the need to provide on each fracture a good quality mesh conforming not only to the traces within the fracture, but also to the other meshes on fractures sharing a trace. If this kind of conformity is required, the meshing process for each fracture is not independent of the others, leading in practice to a demanding computational effort for the mesh generation. In large realistic systems, which can count thousands, or even millions, of fractures, this mesh conformity constraints might lead to the introduction of a very large number of elements, independently of the accuracy required on the solution and possibly leading to over solving, if we consider the level of accuracy of the physical model.

Strategies are proposed in the literature to ease the process of mesh generation and resolution for DFNs of large size. Some authors, see e.g. [4,5], propose a simplification of DFN geometry to better handle the meshing procedure. In other cases, dimensional reduction is explored as in [6,7], where a system of 1D pipes that connect traces with fractures has been used to simplify the problem. Mortar methods are used to relax the conformity condition with fracture meshes, that are only required to be aligned along the traces (see [8,9]).

In the recent paper [10] and follow up works [11,12], the problem of flow in a DFN is retooled as a PDE constrained optimization problem. The approach proposed in these works completely drops the need for any kind of mesh conformity, regardless of trace number and disposition; this goal is attained via the minimization of a given quadratic functional, allowing to obtain the solution for any given mesh. In this framework, any mesh independently generated on each fracture can be used. Since the solution may display a non-smooth behaviour along traces (namely, discontinuous normal derivatives), FEM on meshes not conforming to traces would result in poor solutions in a neighbourhood of the traces. In [10–12] the XFEM is used in order to improve the solution near traces through the introduction of additional non-smooth basis functions, customized for the problem under consideration. The handling of these basis functions requires special care in numerical integration, and might be a source of ill-conditioning due to the possible introduction of almost linearly dependent basis function (see, e.g., [13] and references therein). In the present work the newly conceived Virtual Element Method is in charge for the space discretization on each fracture. Taking advantage from the great flexibility of VEM in allowing the use of rather general polygonal mesh elements, the aforementioned complexities related to XFEM enrichment functions can be avoided. Indeed, a suitable mesh for representing the solution can be easily obtained starting from an arbitrary triangular mesh independently built on each fracture, and independent of the trace disposition. Then, whenever a trace crosses a mesh element, this can be split into two sub-elements obtaining a partial conformity.

All the steps needed for the use of the VEM in conjunction with the optimization approach for DFNs simulations are inherently fracture oriented, and can be executed in parallel. Numerical tests show that this approach leads to an efficient and reliable method.

We remark that the polygonal mesh obtained for VEM discretization naturally paves the way also for the use of a Mortar approach. This possibility is currently under investigation by the authors. Nevertheless, our main target here is to assess the viability of the optimization approach in conjunction with the VEM. Furthermore, within the optimization method, mixing of different discretization strategies (standard finite elements on meshes not necessarily conforming to traces, extended finite elements and virtual elements of different orders) remains possible, thus improving the flexibility to deal with any possible DFN configurations.

The present work is organized as follows: a description of the general problem is provided in Section 2, followed by a brief introduction to the application of virtual element method to the problem at hand in Section 3. Formulation and resolution of the discrete problem are sketched in Section 4. Some technical issues concerning VEM implementation in this context as well as numerical results are given in Section 5. We end with some conclusions in Section 6.

2. Problem description

In this section we briefly sketch the main ideas of the PDE optimization method for discrete fracture network simulations introduced in [10–12].