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High order finite point method for the solution to the sound propagation problems

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Highlights

- The accuracy of the meshfree Finite point method (FPM) was improved.
- The Godunov Riemann solver was used with the polynomial reconstruction.
- Modified FPM was used to solve the acoustic propagation problems.
- The propagation of sound was simulated using the linearized Euler equations.
- High order of the FPM was confirmed by the convergence study.

Abstract

In this paper we present an accuracy improvement of the meshfree Finite point method. This high-order method has been used to solve the sound propagation problems, which can be modelled by linearized Euler equations. High accuracy has been obtained using polynomial reconstruction of variables involved in the Riemann solver. The order of the meshfree method will be verified on 2D acoustic pulse problem which serves as a benchmark problem with known analytical solution.

Keywords: Acoustic pulse problem; Computational aeroacoustics; Finite point method; Linearized Euler equations

1. Introduction

Computational aeroacoustics (CAA) becomes an important research field with the permanent increase of computer performance and accessible memory. The main challenge in CAA is the computation of noise generation and propagation problems using various numerical techniques. One of the possible options is a direct computation from governing partial differential equations (PDEs), but only provided that the underlying numerical methods reach

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high spatial and time accuracy. The reason for this strict requirement is the aerodynamic and acoustic disparity, cf. [1].

The standard solution to the CAA problems is based on numerical methods, which require a fixed mesh or grid, such as finite difference methods [2], finite volume methods [3–5] or finite element methods and their variations [6,7]. An alternative approach represent numerical methods known as meshfree or meshless methods, cf. [8–10], which do not require a predefined mesh. A frequently mentioned advantage of meshfree methods in comparison to the standard mesh-based methods is the absence of time consuming mesh generation. Another, even more important, advantage is their high accuracy. On the other hand, these benefits are offset by additional computational costs. Li, et al., [11] and Wang [12] have recently proposed a meshfree method for CAA based on a radial basis function interpolation. Antunes, cf. [13] has developed a meshfree modification of the method of fundamental solutions.

We have studied the properties of meshfree Finite point method (FPM) proposed by E. Oñate [14–19] to solve the linearized Euler equations (LEE). This hyperbolic system of equations can be seen as a model of sound propagation, cf. [20,21]. We have improved the accuracy of FPM using the polynomial reconstruction of variables in the Riemann solver, [3–5]. A benchmark 2D acoustic pulse problem, proposed by Tam and Webb [2] has been solved using high order FPM. The order of the proposed method has been estimated using a convergence study.

2. Outline

The first part of this paper is devoted to the derivation of 2D LEE. A section describing the FPM in detail follows immediately. An approximation of variables in FPM is performed locally using weighted least squares (WLSQ) method, cf. Section 4.2. The governing PDEs are then collocated at each point in the domain of interest leading to a system of ordinary differential equations (ODEs). A necessary step after semi-discretization is the stabilization of the numerical scheme with respect to the hyperbolic nature of the governing LEE. The accuracy of FPM is strongly related to the accuracy of the directional flux between two neighbouring points. Godunov's method is one of the approximation techniques, that makes it possible to obtain the directional flux by solving the Riemann problem, cf. Section 4.7. By using the more accurate initial condition for the Riemann problem (left and right states), the more accurate solution of the governing equations is obtained. Therefore, a polynomial reconstruction of the left and right states, cf. Section 4.8, is proposed.

The second important part of the paper is devoted to the numerical experiments. A solution to the 2D acoustic pulse problem using high order FPM is presented, cf. Section 5.1. We compare the analytical and numerical solutions computed using different polynomial reconstructions and different space discretizations of the domain of interest, resulting to a convergence study in Section 5.5. A solution to the 2D wall bounded acoustic pulse problem on rectangular and circular geometry is presented in Sections 5.6–5.11.

3. 2D linearized Euler equations

Let us denote the vector function $\mathbf{w}(\mathbf{x}, t) := (\rho(\mathbf{x}, t), u(\mathbf{x}, t), v(\mathbf{x}, t), p(\mathbf{x}, t))^T$ with primitive (physical) variables, i.e. the density, velocity components and pressure, respectively. Therefore, the compressible 2D Euler equations in matrix form read as

$$\frac{\partial \mathbf{w}}{\partial t} + \mathbb{A}_1(\mathbf{w})\frac{\partial \mathbf{w}}{\partial x} + \mathbb{A}_2(\mathbf{w})\frac{\partial \mathbf{w}}{\partial y} = \mathbf{0}, \quad \mathbf{x} = (x, y) \in \mathbb{R}^2,\ t > 0, \tag{1}$$

where the Jacobian matrices of this hyperbolic system are given as follows

$$\mathbb{A}_1(\mathbf{w}) = \begin{pmatrix} u & \rho & 0 & 0 \\ 0 & u & 0 & 1/\rho \\ 0 & 0 & u & 0 \\ 0 & \gamma p & 0 & u \end{pmatrix}, \qquad \mathbb{A}_2(\mathbf{w}) = \begin{pmatrix} v & 0 & \rho & 0 \\ 0 & v & 0 & 0 \\ 0 & 0 & v & 1/\rho \\ 0 & 0 & \gamma p & v \end{pmatrix}, \tag{2}$$

where γ is the adiabatic index ($\gamma = 1.4$ for diatomic gases). The quantities included in $\mathbf{w}$ can be decomposed into a *reference state* (or *mean value*) $\mathbf{w}_0(\mathbf{x})$ and a time dependent *fluctuating* (or *perturbation*) part $\mathbf{w}'(\mathbf{x}, t)$, cf. [1,20,21] in