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# A stable node-based smoothed finite element method for acoustic problems

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## Highlights

- We present a stable node-based smoothed finite element method for acoustic analysis.
- The SNS-FEM can greatly reduce the dispersion error for high wave number problems.
- The present formulation has higher precision and faster convergence rate.
- Higher computational efficiency can be obtained by employing the SNS-FEM.
- The developed methodology works well even for extremely distorted meshes.

## Abstract

It is well-known that the classical "overly-soft" node-based smoothed finite element method (NS-FEM) fails to provide reliable results to the Helmholtz equation due to the "temporal instability". To cure the fatal drawback of NS-FEM and reduce the dispersion error in computational acoustics, this paper proposed a stable node-based smoothed finite element method (SNS-FEM) for analyzing acoustic problems using linear triangular (for 2D space) and tetrahedral (for 3D space) elements that can be generated automatically for any complicated configurations. In the present formulation, the system stiffness matrix is computed using the smoothed acoustic pressure gradients together with the gradient variance items over the smoothing domains associated with nodes of element mesh. It turns out the addition of stabilization term makes the SNS-FEM possess an ideal stiffness, thus successfully cures the temporal instability and significantly reduces the dispersion error in acoustic problems. Numerical examples, including both benchmark cases and practical engineering problems, demonstrate that the SNS-FEM possesses the following important properties: (1) temporal stability; (2) super accuracy and super convergence; (3) higher computational efficiency; (4) insensitive to mesh distortion.



*Keywords:* Acoustic; Numerical methods; Node-based smoothed finite element method (NS-FEM); Temporal stability; Discretization error

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## 1. Introduction

With the increasing concerns on the acoustic performance of enclosed cavities, such as the vehicle passenger compartments and the aircraft cabins, careful considerations must be given in designing these sophisticated products. Currently, the standard finite element method (FEM) [1,2] and boundary element method (BEM) [3,4] are the most reliable and widely-used numerical tools in simulating acoustic scattering with many commercial software packages available, such as MSC Nastran and Virtual Lab. In general, the BEM shows great advantages in solving exterior acoustic problems, since it allows the simulation of fields in unbounded domains; when it comes to interior acoustic problems, acousticians prefer to chose the FEM as the profiler for its simplicity and efficiency.

Finite element method, when applied to time-harmonic wave propagation problems, will inevitably incur solution errors that can be classified into "amplitude error" and "dispersion error" [5,6]. It should be noted that the former is relatively benign since it does not increase with growing wave numbers; the latter, however, accumulates dramatically at higher frequency ranges [1,5]. Acoustic finite element users often believe that a "rule of the thumb" prescribing the minimal discretization per wavelength is sufficient to obtain a reliable result, but this assumption is not always true. In the low frequency range, the FEM works well and can provides very accurate solutions. In the high frequency range, the FEM can no longer give acceptable acoustic pressure predictions unless a sufficiently (far beyond the "rule of the thumb") fine mesh is employed. However, such an extra fine mesh discretization will cause a huge consumption of the computational costs, especially for large scale 3D acoustic problems. Another deficiency encountered by element-based numerical methods is that the accuracy of solutions also depends significantly on the mesh quality and mesh orientation [7]. Mullen and Belytschko [8] pointed out the dispersion error in higher dimension is anisotropic. In order to ensure the accuracy, high quality meshes must be provided when performing the numerical simulation, which inevitably increases the workload in the pre-processing stage.

Studies have shown that the dispersion error in one-dimensional problems can be completely removed by resorting to analytical solutions [1]. However, for higher dimensional acoustic problems, there is no conclusive general theory that can be used to eliminate the discretization error [9]. Over the past several decades, various numerical methods have been proposed by researchers to reduce the pollution error in computational acoustics. They are, (i) the stabilized FEM, including the Galerkin/least squares finite element method (GLS) [10–12], the residual-free bubbles approach (RFB) [13], the multiscale finite element method (MFEM) [14] and the quasi-stabilized finite element method (QSFEM) [15,16]; (ii) higher order finite elements, such as the partition of unity method (PUM) [17,18], the discontinuous enrichment method (DEM) [19] and the p-version FEM [20–22]; (iii) meshless method, including the element-free Galerkin method (EFGM) [23,24] and the radial point interpolation method (RPIM) [25,26]. These numerical improvements are very promising in controlling the pollution error, and significantly improve the calculation accuracy compared with the classical FEM. However, the dispersion error in general two- and three-dimensional acoustic problems still cannot be properly eliminated with these methodologies.

As is known to all, the discretized FEM model based on the standard Galerkin weakform exhibits "overly-stiff" property compared with the exact continuous system, which makes the numerical speed of sound propagates faster than its real value [27–30]. Indeed, this is why the dispersion error increases dramatically at higher frequency range. Therefore, a more effective way in reducing the pollution error is providing a proper stiffness to the discretized system. For this purpose, a stabilized conforming nodal integration (SCNI) approach was first proposed by Chen and his co-workers [31,32]. Later, Liu et al. [33–36] developed a generalized gradient smoothing technique and the so-called G space theory. The authors also proved that the smoothing operation can provides the discretized model a sufficient softening effect, and thus significantly improves the performance of the G space-based numerical methods [37]. Using the node-based strain smoothing technique, a group of node-based smoothed finite element method (NS-FEM) have been proposed by researchers in recent years [38–41]. Studies demonstrate that the NS-FEM performs well in adaptive analysis [42,43], heat transfer analysis [44] and fracture analysis [45]. However, the NS-FEM model exhibits "overly-soft" property which leads to the so-called temporal instability [29,34,35,46]. Hence, the NS-FEM cannot be applied to time-dependent problems (e.g., dynamic problems, transient analysis and acoustic problems) unless other stabilization techniques are incorporated.

In order to eliminate the temporal instability of NS-FEM, various numerical improvements have been proposed in recent years. Generally speaking, these different methodologies can be grossly divided into two groups. The first is the squared-residual stabilization technique, which was first proposed by Beissel and Belytschko [47], and later developed by Zhang et al. [46], Feng et al. [48] and Wang et al. [49]. In which, a stabilization parameter $\alpha$ is introduced to adjust