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Weak coupling for isogeometric analysis of non-matching and trimmed multi-patch geometries

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ABSTRACT

Nitsche's method can be used as a coupling tool for non-matching discretizations by weakly enforcing interface constraints. We explore the use of weak coupling based on Nitsche's method in the context of higher order and higher continuity B-splines and NURBS. We demonstrate that weakly coupled spline discretizations do not compromise the accuracy of isogeometric analysis. We show that the combination of weak coupling with the finite cell method opens the door for a truly isogeometric treatment of trimmed B-spline and NURBS geometries that eliminates the need for costly reparameterization procedures. We test our methodology for several relevant technical problems in two and three dimensions, such as gluing together trimmed multi-patches and connecting non-matching meshes that contain B-spline basis functions and standard triangular finite elements. The results demonstrate that the concept of Nitsche based weak coupling in conjunction with the finite cell method has the potential to considerably increase the flexibility of the design-through-analysis process in isogeometric analysis.

1. Introduction

Isogeometric analysis (IGA) was introduced by Hughes and coworkers [1,2] to bridge the gap between computer aided geometric design (CAGD) and finite element analysis (FEA). The core idea of IGA is to use the same *smooth* and *higher order* basis functions for the representation of both geometry in CAGD and the approximation of solution fields in FEA. Isogeometric analysis turned out to be a superior computational mechanics technology, which on a per-degree-of-freedom basis exhibits increased accuracy and robustness in comparison to standard low-order finite element methods (FEM) [3–5]. IGA has been based on a variety of spline basis functions, e.g. B-splines, non-uniform rational B-splines (NURBS), T-splines [6–8], hierarchical B-splines and NURBS [9–14], PHT-splines [15,16], or LRB-splines [17]. IGA has been successfully applied in a variety of areas, such as structural vibrations [2,18], incompressibility [19–21], shells [22–26], fluid–structure interaction [27–29], turbulence [30,31], phase field analysis [32–35], contact mechanics [36–39], shape optimization [40–42], electromagnetics [43,44], immersed boundary methods [4,11,45–48], boundary elements [7,49], and isogeometric collocation methods [50–52].

From a practical engineering point of view, the paradigm behind IGA is to simplify the cost-intensive mesh generation process required for standard FEA and to support a more tightly connected interaction between CAGD and FEA tools [53–55]. The power and potential of the isogeometric paradigm has been clearly documented in a large number of

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benchmark studies and proof-of-concepts. However, in practice, state-of-the-art large-scale CAGD models rarely consists of single or matching NURBS patches. They are built from dozens or hundreds of trimmed patches that are connected along non-matching boundaries and trimming curves that arbitrarily cut through spline elements. We believe that current IGA technologies need to further evolve in this direction to provide general, robust and efficient design-through-analysis methods that can handle these "dirty" geometries.

In [48,56], we recently presented a weak formulation for the enforcement of essential boundary conditions based on Nitsche's method [57]. We showed that the weak imposition of boundary conditions is particularly suitable in isogeometric analysis and immersed boundary discretizations based on the finite cell method [4,47,48], where non-interpolatory basis functions and boundary conditions cutting through elements obstruct a treatment with the standard strong approach. In [58], we demonstrated the benefits of weakly imposed boundary conditions in structural analysis using the example of a three-dimensional proximal femur bone. In this paper we extend the concept of weak enforcement of boundary conditions to weak enforcement of coupling conditions along interfaces between different B-splines and NURBS discretizations. We are particularly interested in "dirty" interfaces that may include non-matching, overlapping and trimmed B-spline and NURBS patches.

The presented coupling method is based on a Nitsche formulation that ensures variational consistency by introducing flux terms along the coupling interface expressed by primal unknowns of the coupling domains [57,59]. The influence of flux terms that implicitly appear in coupled problems and their beneficial role in analysis was for example shown in [60,61]. Following Nitsche's method, we also apply adequate stabilization terms that depend on material properties as well as the polynomial order and characteristic element size of the discretization. The importance of proper stabilization has been recently shown in [48,60,62–65] for weakly enforced boundary and interface conditions in structural mechanics, fluid mechanics and fluid–structure interaction problems. A further important ingredient of our approach is the fictitious domain concept that we apply to mitigate the influence of trimmed parts of B-spline and NURBS patches. We apply the NURBS version of the finite cell method [4,11,48,66,67] that uses composed Gauss quadrature to resolve inter-element trimming curves and interfaces. Its major advantage is its flexibility that allows the treatment of very complex boundary and interface geometries with a high degree of automatization. The quadrature based approach of the finite cell method can be interpreted in the context of trimmed NURBS geometries as the local shift of the geometry resolution from the basis functions to the integration point level.

We believe that our methodology has the potential to significantly increase the flexibility and ease of the design-through-analysis process in isogeometric analysis. We show that our methodology does not compromise accuracy, achieving optimal rates of convergence for uniform mesh refinement and exponential rates for p-refinement. We also test the influence of the stabilization terms on the solution quality for various discretizations and provide guidelines for the optimal choice of the stabilization parameter. We demonstrate the main properties of our approach and its applicability by a range of numerical examples in two and three dimensions that include non-matching interfaces, trimmed NURBS geometries and the connection of spline discretizations with standard triangular finite element meshes.

The structure of the paper is as follows: Section 2 briefly reviews B-spline and NURBS discretizations and the trimming concept in the context of isogeometric analysis. Section 3 presents in detail the variational formulation and discretization of the weak coupling terms based on Nitsche's method and the finite cell method. In particular, we provide a summary of the mathematical background for the optimal choice of the stabilization parameter. Section 4 shows a set of examples that demonstrate the performance of our methodology and its significant potential for simplifying the design-though-analysis process. Section 5 summarizes our most important points and motivates future research in this direction.

2. Isogeometric analysis: spline discretizations and the trimming concept

We start with a concise introduction to isogeometric analysis with particular emphasis on B-spline and NURBS basis function technology and the trimming concept that is widely used in geometric modeling based on CAGD. We also provide the variational formulation of elasticity in a multi-domain context and review the various coupling terms that emerge at the domain interfaces.

2.1. B-spline and NURBS patches

Following the isoparametric concept of standard FEM, isogeometric analysis uses higher order smooth spline functions, in particular B-splines and non-uniform rational B-splines (NURBS). According to the isogeometric paradigm, these functions are used to represent both the geometry in CAGD and the approximation of the solution field in the analysis. B-splines and NURBS form the basis technology in today's state-of-the-art CAGD tools and allow the representation of arbitrary free-form surfaces.

A univariate B-spline function of polynomial degree p is specified by n basis functions $N_{i,p}(\xi)$, $(i = 1, \ldots, n)$ in the parametric space ξ. The non-decreasing set of $(n + p + 1)$ coordinates ξ_i are the so-called knots and subdivide the parameter space into $(n + p)$ knot spans forming a patch [1,2].

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\} \tag{1}$$