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Acoustic isogeometric boundary element analysis

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ABSTRACT

An isogeometric boundary element method based on T-splines is used to simulate acoustic phenomena. We restrict our developments to low-frequency problems to establish the fundamental properties of the proposed approach. Using T-splines, the computer aided design (CAD) and boundary element analysis are integrated without recourse to geometry clean-up or mesh generation. A regularized Burton–Miller formulation is used resulting in integrals which are at most weakly singular. We employ a collocation-based approach to generate the linear system of equations. The method is verified against closed-form solutions and direct comparisons are made with conventional Lagrangian discretizations. It is demonstrated that the superior accuracy of the isogeometric approach emanates from the exact geometric description encapsulated in the T-spline. The method is then applied to a real-world application to illustrate the potential for integrated engineering design and analysis.

1. Introduction

Boundary element methods (BEM) are frequently used to simulate acoustic phenomena. The method's popularity stems from two primary considerations:

1. BEM is a mesh reduction technique. In other words, volumetric problems can be solved on surfaces. This dramatically reduces the complexity of the mesh generation process.
2. BEM incorporates boundary conditions posed over infinite domains. For example, for exterior acoustic problems, the Sommerfeld radiation condition [1] at infinity is included naturally without resorting to techniques based on truncated domain discretizations [2–5].

The initial use of BEM to solve Helmholtz problems for arbitrarily shaped bodies can be traced to [6–10] where simple discretization procedures, based on constant and linear elements, were applied. The majority of these formulations used a collocation-based approach while Galerkin BEM has recently gained some popularity [11–14].

To solve exterior radiation and scattering problems using integral equations, instabilities arise when the wavenumber coincides with the eigenmodes of the corresponding interior problem. This leads to spurious results. In the literature, there are three primary solutions to this problem:

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1. The CHIEF approach [15] places a certain number of additional collocation points at interior points resulting in an over-determined system of equations. The advantage of this approach is the relative ease of implementation. Unfortunately, the exact location and number of additional points that must be used is not known *a priori* and the condition number of the resulting linear system suffers.
2. The Burton–Miller (BM) formulation [16] combines an additional boundary integral equation with the original boundary integral equation. The resulting linear combination is stable. Difficulties arise during implementation, however, since hypersingular integrals must be evaluated which impose additional continuity constraints on the approximation of the acoustic potential.
3. The Green's function is modified in such a way as to render it stable [17,18]. This approach is only valid for a restricted range of wavenumbers.

Due to its robustness and rigorous mathematical foundation, the BM approach is chosen in the present study. As shown in Section 5, the hypersingular nature of the formulation can be reduced to that of a weakly singular formulation by an appropriate regularization technique. This is possible due to the smoothness inherent in T-spline discretizations.

Recent work on the acoustic boundary element method has focused on solving high-frequency problems [19–23,14,24]. Much attention has also been given to acceleration strategies such as the fast multipole method (FMM) [25] applied to the Helmholtz equation [26–28] and hierarchial matrices and adaptive cross approximation techniques [29,30]. These methods are most commonly used to solve problems posed over extremely fine discretizations of three-dimensional geometries.

Despite the obvious link with CAD surface technology little attention has been given to establishing a fully integrated BEM/CAD solution. Several early efforts applied B-splines and BEM to solve Laplace problems [31–35]. Later, a stronger link with CAD was established through the so called "NURBS panel method" [36–38]. Recently, several authors have used NURBS for geometry representation while using a Lagrangian basis for approximation [39,40].

The successful coupling of T-spline CAD surfaces, isogeometric analysis, and BEM has demonstrated that a completely integrated BEM/CAD technology is feasible [41]. In isogeometric analysis, the geometric basis is used as the finite element basis [42]. In the context of surface-based analysis technologies like BEM, the mesh generation and geometry clean-up steps are eliminated if the underlying CAD geometry is analysis-suitable. In this way, the exact geometric description is embedded in the analysis. Research efforts in isogeometric BEM are expanding rapidly [43–49].

Although predominant in CAD, NURBS descriptions of geometry are often not analysis-suitable. Complex NURBS models are comprised of many NURBS patches which are usually discontinuous across patch boundaries. Due to the tensor product nature of NURBS local refinement is also not possible, a particularly vexing shortcoming from an analysis standpoint [50]. T-splines [51–53] overcome these shortcomings. Complex T-spline geometry is described by a single watertight surface. Efficient procedures also exist for T-spline local refinement [54–56]. Additionally, NURBS are a proper subset of T-splines which make T-splines fully compatible with NURBS-based technology. These properties make T-splines an ideal partner for isogeometric boundary element methods. In the context of isogeometric analysis T-splines have been successfully applied in various settings including elasticity [50,54], shells [57], fluid–structure interaction [58], electromagnetics [59], fracture and damage [60–62], and contact [63].

2. Structure, content and notation of the paper

This paper is structured as follows. First, the conventional Helmholtz boundary integral equation is presented, the Burton–Miller approach, required for a stable formulation of exterior problems, is described, and the use of a regularization technique to reduce all integrals to at most, weakly singular, is formulated. A brief introduction to T-spline discretizations is then presented. Finally, the method is applied to several three-dimensional acoustic problems. A direct comparison is made with conventional Lagrange BEM discretizations. All problems considered in the present work are restricted to low frequencies with no modifications made to basis functions to account for the oscillatory nature of acoustic solutions. High frequency problems are excluded in this study and form a subject for future research.

We adopt the notation presented in [41] in this paper. The following definitions are used: d is the polynomial degree (which, for T-splines, is equal to three), $i,j = 1,2,3$ are indices for the spatial components of vectors and tensors, n is used to denote the number of global T-spline basis functions, n_{el} is the number of elements, and n_{en} is the number of non-zero T-spline basis functions over a particular element e. Uppercase indices imply a global index, while lowercase indices imply a local index. The present study is focused entirely on three-dimensional problems. **Boldfont** letters are used to indicate vectors and matrices where dimensions are implied.

3. The Conventional Boundary Integral Equation for Helmholtz problems

Given a domain $\Omega \subset \mathbb{R}^3$ with boundary $\Gamma \equiv \partial\Omega$, the boundary integral equation for the Helmholtz equation is defined as

$$C(\mathbf{s})\phi(\mathbf{s}) + \fint_\Gamma \frac{\partial G(\mathbf{s},\mathbf{x})}{\partial n}\phi(\mathbf{x})\,\mathrm{d}\Gamma(\mathbf{x}) = \int_\Gamma G(\mathbf{s},\mathbf{x})\frac{\partial \phi(\mathbf{x})}{\partial n}\,\mathrm{d}\Gamma(\mathbf{x}) \tag{1}$$