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A consistent 3D corotational beam element for nonlinear dynamic analysis of flexible structures

Thanh-Nam Le [a,b], Jean-Marc Battini [b,*], Mohammed Hjiaj [a]

[a] Université Européenne de Bretagne, INSA de Rennes, LGCGM/Structural Engineering Research Group, 20 avenue des Buttes de Coësmes, CS 70839, 35708 Rennes Cedex 7, France
[b] KTH, Royal Institute of Technology, Department of Civil and Architectural Engineering, SE-10044 Stockholm, Sweden

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ABSTRACT

The purpose of the paper is to present a corotational beam element for the nonlinear dynamic analysis of 3D flexible frames. The novelty of the formulation lies in the use of the corotational framework (i.e., the decomposition into rigid body motion and pure deformation) to derive not only the internal force vector and the tangent stiffness matrix but also the inertia force vector and the tangent dynamic matrix. As a consequence, cubic interpolations are adopted to formulate both inertia and internal local terms. In the derivation of the dynamic terms, an approximation for the local rotations is introduced and a concise expression for the global inertia force vector is obtained. To enhance the efficiency of the iterative procedure, an approximate expression of the tangent dynamic matrix is adopted. Four numerical examples are considered to assess the performance of the new formulation against the one suggested by Simo and Vu-Quoc (1988) [48]. It was observed that the proposed formulation proves to combine accuracy with efficiency. In particular, the present approach achieves the same level of accuracy as the formulation of Simo and Vu-Quoc but with a significantly smaller number of elements.

1. Introduction

Flexible beams are used in many applications, for instance large deployable space structures, aircrafts and wind turbines propellers, offshore platforms. These structures undergo large displacements and finite rotations, but the strains remain small. Their nonlinear dynamic behavior is often simulated using geometrically nonlinear spatial beam elements. In the literature, several approaches have been used to develop such elements. A large number of them have been formulated in the total Lagrangian context [4,9,10,19,26,29,31,32,36,38,48,52,53]. An other attractive way to develop effective nonlinear dynamic beam elements, which is adopted in the present work, is to use the corotational approach. Indeed, this framework has been adopted by several authors to develop efficient beam and shell elements for the nonlinear static and dynamic analysis of flexible structures. [1,6,8,11,13–18,21,22,30,39,41,43,50]. Several versions of the corotational method have been proposed in the literature. The one employed in this work has been proposed by Rankin and Nour-Omid [40,44], and then further developed by Battini and Pacoste [5]. The main idea of the method is to decompose the motion of the element into rigid body and pure deformational parts. During the rigid body motion, a local coordinates system, attached to the element, moves and rotates with it. The deformational part is measured in this local system. The main interest of the approach is that different assumptions can be made to represent the local deformations, giving rise to different possibilities for the local element formulation.

* Corresponding author.
E-mail addresses: Thanh-Nam.Le@insa-rennes.fr (T.-N. Le), jean-marc.battini@byv.kth.se (J.-M. Battini), mohammed.hjiaj@insa-rennes.fr (M. Hjiaj).

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For the geometrically and materially static nonlinear analysis of beam structures with corotational approach, several local formulations have been proposed by Battini and Pacoste [5], and Alsafadie et al. [2]. The results of a comparative study of 3D beam formulations, which can be found in [2], have shown that local beam elements based on cubic interpolations are more efficient and accurate than the ones which employ linear interpolations. However, in dynamics, one has to deal with the inertia terms which by nature are complicate to formulate. This is particularly true in the corotational formulation of Bernoulli-type beam elements. This difficulty has hampered the development of the corotational approach in nonlinear dynamics. To avoid the consistent derivation of the inertia terms, several routes have been considered.

For 2D dynamic analysis, quite a few authors [39,41,50] have adopted the constant lumped mass matrix without any attempt to check its accuracy. Iura and Atluri [30] suggested to simply switch to a Timoshenko beam model where the mass matrix is constant and therefore the inertia terms are simple to evaluate. Behdinan et al. [8] proposed a 2D corotational dynamic formulation where cubic interpolations have been used to describe the global displacements, which is not consistent with the idea of the corotational method as originally introduced by Nour-Omid and Rankin [40]. More recently, Le et al. [34] developed a consistent 2D corotational beam element for nonlinear dynamics. Cubic interpolations were used to describe the local displacements and to derive both inertia and internal terms. Numerical results demonstrated that the formulation is more efficient than the classic formulations (i.e., with the constant Timoshenko and the constant lumped mass matrices). For 3D dynamic analysis, Crisfield et al. [16] suggested to use a constant Timoshenko mass matrix along with local cubic interpolations to derive the internal force vector and the corresponding tangent stiffness matrix. As pointed out by Crisfield et al. [16], this combination is not consistent but it provides reasonable results when the number of elements is large enough. Hsiao et al. [22] presented a corotational formulation for the nonlinear analysis of 3D beams. However the corotational framework adopted in [22] is different from the classic one as proposed by Nour-Omid and Rankin [40] and adopted in this paper.

The objective of this paper is to extend the consistent 2D corotational dynamic formulation presented in [34] to 3D beam structures. Hence, the corotational framework (i.e., the decomposition into rigid body motion and pure deformation) is used to derive not only the internal force vector and the tangent stiffness matrix but also the inertia force vector and the tangent dynamic matrix. The element has two nodes and is initially straight. The same cubic interpolations are adopted to formulate both inertia and internal local terms. In doing so, the complex expressions of the inertia terms are significantly simplified by adopting a proper approximation for the local rotations. To enhance the efficiency of the iterative procedure, the less significant term in the tangent dynamic matrix is ignored (see [35]).

Regarding the time-stepping scheme, the classic HHT α method (with $\alpha = -0.05$) is adopted in this work. This energy-dissipative method, which is implemented in several commercial FEM programs (Abaqus, Lusas) and was employed by many authors [10,16,32], limits the influence of high frequencies by introducing a numerical damping. The latter often avoids numerical instability, but also results in dissipation of the total energy. It can be noted that for beam structures, more robust alternatives to the HHT α method have been proposed in the literature [27].

Four numerical examples are presented with the objective to compare the performances of the new formulation against two other approaches. The first approach is similar to the one presented above, but linear local interpolations, instead of cubic ones, are used to derive only the dynamic terms. The purpose is to evaluate the influence of the choice of the local interpolations on the dynamic terms. The second approach is the classic total Lagrangian formulation proposed by Simo and Vu-Quoc [46–48].

The paper is organized as follows. Section 2 presents some aspects of the parametrization of finite rotations that are essentials for the subsequent developments. Section 3 is devoted to the corotational beam kinematics, whilst the local beam element formulation is introduced in Section 4. The internal force vector and the tangent stiffness matrix are briefly presented in Section 5. More details about the derivation can be found in [5]. Sections 6 and 7 focus on the derivation of the inertia terms and the time stepping method. In Section 8, four examples are presented in order to assess the accuracy of the present dynamic formulation. Finally, conclusions are given in Section 9.

2. Parametrization of finite rotations

Due to the complex nature of 3D finite rotations, the parametrization of the finite rotations is a central issue in the development of 3D beam elements. In the literature, many ways to parameterize the finite rotations have been proposed, e.g.: spin variables, rotational vector, Euler parameters, Rodrigues parameters. In the current work, spatial spin variables and spatial rotational vector are used.

In this section, the relations required for the development of the present formulation are introduced. For a more complete description of the finite rotations, the reader is referred to textbooks and papers such as [3,7,12,20,23,25,33].

The coordinate of a vector $\mathbf{x}_o$ that is rotated into the position $\mathbf{x}$ (see Fig. 1) is given by the relation

$$\mathbf{x} = \mathbf{R}\,\mathbf{x}_o. \tag{1}$$

Due to its orthonormality, the rotation matrix $\mathbf{R}$ can be parameterized using only three independent parameters. One possibility is to use the rotational vector defined by

$$\boldsymbol{\theta} = \theta\,\mathbf{n}, \tag{2}$$