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Imposing boundary and interface conditions in multi-resolution wavelet Galerkin method for numerical solution of Helmholtz problems

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Abstract

In this paper, we propose a new formulation for numerical solution of boundary value problems using mesh-free multi-resolution wavelet Galerkin (WG) method. A difficulty with the WG method is imposing boundary condition constraints. Single scale wavelet basis functions together with a jump function approach using cubic polynomial functions have been used in solving some boundary problems in the literature. Ramp and cubic polynomial functions have been also used in the jump function approach described in element-free Galerkin context, and it is proven ramp functions exactly satisfy the desirable property for the derivative of jump, but the cubic polynomial functions do not. In this paper, a multi-resolution WG method is employed, and both ramp and cubic polynomial jump functions are applied. Obtained results in one dimension are compared with analytical solutions. Results obtained with ramp jump functions in two dimensions are also reported. The results obtained with the WG method manifest boundary and interface conditions are accurately imposed when the ramp jump functions are applied. Numerical results are in good agreement with accurate solutions.

Keywords: Mesh free method; Wavelet method; Multi-resolution analysis; Galerkin method

1. Introduction

The Helmholtz equation is used to describe a couple of different physical problems (e.g. electromagnetic fields, acoustics and transmission problems [1,2]) that must be resolved analytically or numerically to obtain the solution. The finite element method (FEM) is a conventional numerical approach that has been successfully applied to various boundary value problems in science and engineering such as fluid dynamics [3], wave

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propagation problems [4–7], and electromagnetic fields [8–11]. This method relies on the primary idea of replacing a continuous function over the entire solution domain by a piecewise continuous approximation, typically using polynomials, over a set of sub-domains called finite elements. FEM requires a structure of interconnected elements via nodes called finite element mesh which is considered as a constraint in some applications [12–15]. For example, in an inverse problem involving moving objects or objects with changing geometrical appearance, when FEM is applied to solve partial differential equations for providing simulated data, mesh distortion is inevitable and susceptible to produce error in numerical results.

The mesh free (MF) methods are effective in problems with variable geometries or mechanical movements; since they are purely based on nodes distribution. This eliminates all problems related to the mesh shape or size [12,15–18]. So far, different MF methods have been proposed to solve some field related engineering problems. The element free Galerkin (EFG) [14,15] and wavelet techniques [19–35] are well known mesh-free methods. The EFG technique is considered as a MF method, because it requires only a set of nodes distributed over the entire solution domain. It is used for solving partial differential equations with the help of shape functions coming from moving least squares approximation. Since the EFG shape functions do not satisfy the Kronecker delta criterion, boundary and interface conditions should be applied by especial numerical techniques such as employing jump functions [36]. For the numerical implementation of this method, it is necessary to calculate the local summation in the vicinity of nodes. Wavelet based mesh-free techniques have been developed for solving partial differential equations in the two different forms: the wavelet collocation [19–22] and the wavelet Galerkin (WG) [23–35] methods. In the wavelet collocation method, every wavelet is uniquely related to a collocation point and the solution is yielded in physical space on a computational grid. In the WG method, the problem is solved for wavelet coefficients of function. In contrast to the EFG and wavelet collocation techniques, the WG method requires no local summation in the vicinity of nodes in numerical implementations. In the inverse problem, using a set of fixed nodes inside the domain including moving objects may introduce errors in numerical results.

In the conventional WG approach, the Galerkin formulation is built based on calculating integrals with integrands being products of wavelet basis functions and their derivatives; these integrals are called connection coefficients. The higher order derivatives of compactly supported wavelets are highly oscillatory, which may cause large variations in these connection coefficients, and produce instability in numerical implementation [33,34,37]. Therefore, the WG method is difficult in numerical implementation and needs additional algorithms and techniques for computing the exact values of connection coefficients [38,39]. Furthermore, to resolve the instability issue, some reduced WG formulations have been described in [27,34]. In these reduced formulations, the connection coefficients are calculated based on the first derivative of the wavelet basis functions.

In order to apply boundary conditions in WG method with bounded solution domain, three different techniques: periodized wavelets [23–25], fictitious domains method [26–31] and wavelet based finite element approach [32–35] have been employed. In the periodized wavelets, the applications of the WG method are limited to the case where it is assumed the boundary conditions are periodic. In the fictitious domains method, the solution domain is extended to a larger fictitious domain with periodic boundary conditions and the original boundary conditions are applied by enforcing suitable additional conditions [26,30]. However, this method also has some limitations such as the coefficient matrices are usually ill-conditioned because the number of unknown coefficients is larger than the number of equations; extending the solution domain increases the computational complexity and cost, and the approximation may be oscillated near the boundaries [26,30,31]. In the wavelet based finite element approach, the boundary conditions are enforced by utilizing some specific algorithms for calculating connection coefficients in the bonded domains [35,40,41]. It should be noted that, in this approach, solutions depend on the shape and size of applied domains. Moreover, for the same reason as EFG, wavelet based finite element approaches [23,34] employ jump functions for applying boundary and interface conditions [34]. Two following properties for a desirable jump function a) the jump function has compact support, and b) the derivative of jump function has the form of a Heaviside step function, are considered in [36]. It has been proven in [36] that ramp functions exactly satisfy the second property, but the cubic polynomial functions do not.

In [16–18,34], a set of single scale wavelet basis functions have been used for realizing the wavelet based approximation in electromagnetic field applications. To apply the boundary and interface conditions cubic