Available online at www.sciencedirect.com

ScienceDirect

Comput. Methods Appl. Mech. Engrg. 276 (2014) 312–340

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Event-driven integration of linear structural dynamics models under unilateral elastic constraints

Alexandre Depouhon [a,b], Emmanuel Detournay [a], Vincent Denoël [b,*]

[a] Department of Civil, Environmental and Geo-Engineering, University of Minnesota, USA

[b] Departement of Architecture, Geology, Environment and Constructions, Structural Engineering Division, University of Liège, Belgium

Received 9 January 2014; received in revised form 16 March 2014; accepted 24 March 2014
Available online 1 April 2014

Abstract

This paper proposes an algorithm for the numerical simulation of linear structural dynamics problems under unilateral elastic constraints, *i.e.*, constraints with a linear force/displacement characteristic whenever active. The presented procedure relies on an event-driven strategy for the handling of the contact constraints, in combination with one-step schemes dedicated to the time integration of the second-order equations of motion. Efficiency of the procedure follows from the use of cubic Hermite interpolation to continuously extend the normal gap functions that reflect the openings of the contact interfaces. Robustness follows from the proper handling of complex numerical situations, *e.g.*, numerical grazing or discontinuity sticking, through appropriate algorithm structure and numerical implementation. And, integration stability is guaranteed by the very nature of the algorithm and that of the one-step integration scheme. Following a detailed coverage of the integration procedure and the countermeasures to the expected numerical difficulties, three application examples are treated for illustration purposes. A MATLAB implementation of the procedure is provided online; download and usage information are given in Appendix A.

Keywords: Structural dynamics; Unilateral constraints; Continuous extension; Cubic Hermite interpolation; Discontinuity sticking; Numerical grazing

* Corresponding author. Address: University of Liege, Department of Architecture, Geology, Environment and Constructions, Structural Engineering Division, Chemin des Chevreuils, 1, B52/3, 4000 Liege, Belgium. Tel.: +32 (0) 43662930; fax: +32 (0)4 366 91 33.
E-mail address: v.denoel@ulg.ac.be (V. Denoël).

http://dx.doi.org/10.1016/j.cma.2014.03.013
0045-7825/

0. Nomenclature

Conventions apply throughout the paper.

- Vectors and matrices are denoted by bold lower and upper case characters, *e.g.*, $\mathbf{v} \in \mathbb{R}^m = \mathbb{R}^{m\times 1}$ and $\mathbf{M} \in \mathbb{R}^{m\times n}$.
- Indices i, j are used to denote component extraction from vectors and matrices
  - v_i refers to the i-th entry of vector $\mathbf{v}$;
  - $\mathbf{M}_i$ denotes the i-th column of matrix $\mathbf{M}$;
  - M_{ij} indicates the scalar entry of matrix $\mathbf{M}$ located at row i and column j.
- Indices k, n indicate indexation with respect to time, *i.e.*, $x_k := x(t_k)$, $x_n := x(t_n)$.
- Indices a, b refer to body identification in multibody systems.
- Calligraphic letters are used to denote compact integer sets. For instance, the set of indices 1 to q is referred to as $\mathcal{C} = \{1, \ldots, q\}$.
- We write the vertical concatenation of column vectors as $\mathbf{x} = [\mathbf{a}; \mathbf{b}]$.

1. Introduction

The development of methods for the numerical handling of unilateral constraints in a dynamic setting is of the highest interest for modeling today's complex engineering applications. Fruits of intense research activities in computational mechanics, several approaches have been proposed. Each of them aims at guaranteeing the quality/accuracy, the robustness and the stability of the integration procedure, in the most efficient manner. Two main families of methods are designed for the handling of unilateral constraints [1]: formulations that exactly enforce the non-interpenetration condition between contacting surfaces, *e.g.*, methods relying on Lagrange multipliers [2] or time-stepping schemes [3], and penalty-based methods that relax it by introducing a constitutive contact model relating the interpenetration to the contact force [4]. Covering both families, Doyen et al. [5] propose a review of several methods, as applied to the dynamic Signorini problem; that is, the 1-dimensional longitudinal impact of an elastic bar against a rigid wall.

This paper is concerned with the problem of handling unilateral elastic constraints in combination with linear dynamics problems (but for the contact interaction). Such constraints arise, for instance, from the use of a quadratic potential to penalize body interpenetration. They also show up in the modeling of percussive drilling—the main driver to this research—to represent the bit/rock interaction law [6–8]. However, in the latter problem, their (de)activation is not solely driven by the normal gap (distance between the contact interfaces), but also by functions depending on the velocity field [8]. With that application in mind, event-driven integration imposes itself as the *ad hoc* integration procedure. Although it can be interpreted as a form of active set strategy for the handling of contact constraints, the use of event-driven schemes is nonetheless more to be found in the fields of non-smooth dynamics and hybrid (switched) systems rather than in computational contact mechanics [9–11].

With a growing interest for these disciplines over the last decades, substantial contributions have been made on the theoretical and numerical aspects, *e.g.*, the pioneering work of Filippov [12] on discontinuous ordinary differential equations (ODEs) or the extensive review tutorial on hybrid dynamical systems by Göbel et al. [13]. This growing research momentum has also led to the development of software simulation packages, such as INRIA's SICONOS simulation framework for the time integration of mechanical systems subject to unilateral contacts and Coulomb friction [14] or the Computational Continuation Core (COCO) for the continuation of non-smooth problems developed by Dankowicz and co-workers [15]. However, to the authors' knowledge, there are no dedicated algorithms or software packages for the time integration of linear structural dynamics problems under unilateral elastic constraints that can be easily extended to trigger functions that depend on the velocity field, which motivates the present research work.

Particular to non-smooth systems is the occurrence of events along the system trajectory. At these specific points, the trajectory loses smoothness and can even exhibit discontinuities in the presence of reset maps [16, Chap. 2], thereby yielding a piecewise smooth trajectory. Examples of such events are, for instance, the