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Structural topology optimization with strength and heat conduction constraints

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Abstract

In this research, a topology optimization with constraints of structural strength and thermal conductivity is proposed. The coupled static linear elastic and heat conduction equations of state are considered. The optimization problem was formulated; viz., minimizing the volume under the constraints of p-norm stress and thermal compliance introducing the qp-relaxation method to avoid the singularity of stress-constraint topology optimization. The proposed optimization methodology is implemented employing the commonly used solid isotropic material with penalization (SIMP) method of topology optimization. The density function is updated using sequential linear programming (SLP) in the early stage of optimization. In the latter stage of optimization, the phase field method is employed to update the density function and obtain clear optimal shapes without intermediate densities. Numerical examples are provided to illustrate the validity and utility of the proposed methodology. Through these numerical studies, the dependency of the optima to the target temperature range due to the thermal expansion is confirmed. The issue of stress concentration due to the thermal expansion problem in the use of the structure in a wide temperature range is also clarified, and resolved by introducing a multi-stress constraint corresponding to several thermal conditions.

Keywords: Topology optimization; Stress constraints; Heat conduction; Thermal expansion; Sensitivity analysis

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1. Introduction

Static strength and heat conduction are two important issues in the design of mechanical structures. When strength is insufficient to support an applied load, a structure can suffer serious damage or break completely in the worst case. When heat conduction is insufficient for heat to **dissipate** from a certain heat source, the temperature may increase until there is injury to users or damage to surrounding devices. These criteria must sometimes be discussed together for one mechanical part. For example, automotive engine blocks are a typical structure requiring (1) strength supporting the load generated by an explosion and loads from mechanical movements of the internal and external parts and (2) heat conduction to release heat from an explosion to the air for the sake of efficient running. On the other hand, recent small electric devices containing central processing units, which can be a serious heat source, have similar requirements. Because of the limited space of such a device, the shell must perform the structural role of supporting the electric part from external loads and the role of a heat sink to release heat generated by the central processing unit into the open air simultaneously. Moreover, the two performance criteria are closely related through the phenomenon of thermal expansion.

Recently, topology optimization (TO) [1,2] has greatly assisted the development of novel mechanical structures because it enables fundamental shape optimization even for a complicated physical problem in topology. The strength of a structure is usually evaluated as a nominal stress that must not exceed a certain limit. To prevent these failures, a practical engineering approach is to calculate the nominal stress values of a structure of interest employing the finite element method (FEM) and to confine them to a certain maximum value by changing the geometry or the material of the structure.

Constructing a TO that minimizes volume subject to stress constraints has been regarded as a very difficult problem for many reasons, including the singularity issue and the local behavior of the stress constraints [3–16]. First, according to previous research, discontinuities arise when a design variable of the solid isotropic material with penalization (SIMP) method converges to zero to simulate non-structural regions ("void" regions). When the design variables of some finite elements converge to zero, the stresses of the corresponding elements converge to finite values. If the stress of these elements reaches the specified stress limit, the elements will remain as a structural member to satisfy the stress constraint. From a structural point of view, however, the stress values of the finite elements simulating the non-structural regions should be zero; i.e., no structure and no stress. Thus, the global optima can be obtained only by eliminating such elements. A local optimal topology with members violating the stress constraint is called a singular optimum. Such singularity of optima in TO was first observed by Sved and Ginos [17] using a simple three-bar truss example under multiple loading conditions. Cheng and Jiang [18] then determined that the fundamental reason for this observation were the discontinuities of the stress constraints at the zero cross-sectional area. There have been many solutions and relaxation methods proposed to avoid singular optima and obtain a global optimum numerically such as the epsilon relaxation method [4,5], the qp-relaxation method [7,8], and the relaxed stress indicator method [10]. Introducing TO methodology without intermediate density like the level-set method or evolutionary TO is also an effective way to avoid singular optima [6,11].

Second, as the nominal stress values of all finite elements of interest must be constrained, from a computational point of view there are too many constraints to efficiently solve the optimization problem with a dual optimizer. As the computational cost for sensitivity analysis and sub-optimization increases, one must resort to approximation methods and other remedies. One such method is the constraint selection method, which selects only active stress constraints and calculates their sensitivity values. Recently, methods of representing a stress measure (sometimes called a global stress measure) have been proposed [9,10,14,16]. In these approximation methods, rather than considering all the constraints, one or several global constraint functions indirectly reflecting the behaviors and effects of the locally defined stress constraints are used. As only some constraint values and the corresponding sensitivity values are calculated, it becomes important to choose an appropriate form for the approximated global stress measure. Until now, the proposals have been the p-norm approach or the Kreisselmeier–Steinhauser (KS) approach [9,10], a global stress measure based on boundary curvatures [14] and a global stress measure based on stress gradients [14,16]. Moreover, the stress criterion represented as a functional over the whole domain is useful for sensitivity analysis using the adjoint method [19].