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A Residual Stress Decomposition based Method for the Shakedown analysis of structures

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Abstract

In the attempt to extend the life of a structure, or a component, which is subjected to cyclic loading history it is important to provide safety margins against excessive inelastic deformations. Direct methods and in particular shakedown analysis provide the only means towards this end. Most of the approaches in shakedown analysis are based on the two theorems of plasticity and are tuned with the solution of optimization algorithms. In this paper a new method is presented which approaches the problem in a different way. The method makes use of a recently published direct method, called the Residual Stress Decomposition Method (RSDM), which assumes the decomposition of the residual stresses into Fourier series in time. The RSDM may predict any cyclic elastoplastic state for a given cyclic loading history. With the present approach a loading of prescribed limits is converted to an equivalent loading which has a prescribed time history. The procedure approaches the shakedown loading from above starting from a loading factor that makes the whole structure plastic. The procedure generates a descending sequence of loading factors which shrinks the load domain until the only remaining term of the Fourier series is the constant term. It is formulated within the finite element (FE) method and an elastic-perfectly plastic material with a von Mises yield surface is assumed. It may be directly implemented in any FE code. The versatility of the approach is shown through examples of application.

Keywords: Direct methods; Plasticity; Shakedown analysis; Cyclic loading; Limit cycle; Fourier series

1. Introduction

The high level of loadings, that most civil and mechanical engineering structures are subjected to, force them to develop irreversible strains, such as plastic strains. For civil engineering structures, like bridges, pavements, buildings, and offshore structures, such typical loadings are heavy traffic, earthquakes or waves.

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On the other hand, the coexistence of thermal and mechanical loadings on mechanical engineering structures, like, for example, nuclear reactors and aircraft propulsion engines, leads them also to stress regimes well beyond their elastic limit.

When the exact loading history is known, one may estimate the long term behavior of a structure on the basis of cumbersome time stepping calculations. A much better alternative, that requires much less computing time, is offered by the direct methods that may predict whether, under the given loading, the structure will become unserviceable due to collapse or excessive inelastic deformations. Moreover, it very often happens that the complete time history of loading is not known, but only its variation intervals. In these cases, direct methods are the only way to establish safety margins.

Based on the fact that for structures made of stable materials [1] an asymptotic state exists, direct methods try to estimate this state right from the start of the calculations. Typical examples of such methods are the limit analysis for monotonic loading and the shakedown analysis for loading varying cyclically. For small displacements and elastic-perfectly plastic solids the search for an elastic shakedown state is normally based on the lower bound [2] or the upper bound [3] theorems.

Since these two seminal works, many extensions were made to include other effects, for example, geometric nonlinearities (e.g. [4–6]). Conditions for extending the static theorem to elastic-perfectly plastic cracked bodies have been presented in [7].

Concerning extensions of elastoplastic material behavior, various researchers have also studied limited linear (e.g. [8,9]) and nonlinear kinematic hardening (e.g. [10–12]). Recent developments on the subject have appeared in [13].

Non associated plasticity has also been addressed (e.g. [14–17]). Furthermore, shakedown theorems have also been formulated within the framework of gradient plasticity concepts [18]. Finally, non-stationary loads have also been considered (e.g. [19,20]).

Most of the numerical approaches towards the solution of the shakedown problem are based on either the lower or the upper bound theorems. They are cast in the form of mathematical programming (MP) aiming to minimize or maximize an objective function which normally represents the loading factor. Depending on whether the objective function and/or the constraints are linear or nonlinear the problem can be formulated as a linear (LP) (e.g. for early works [21,22]) or a nonlinear (NLP) programming problem (e.g. for an early work [11]). The discretization of the continuum by a large number of finite elements and the big number of constraints often lead to the solution of large size optimization problems. To solve these problems various numerical techniques have been developed. One may mention here the reduced basis technique (e.g. [23,24]) or algorithms based on Newton iterations (e.g. [25,26]). The optimization problem was solved in [27] based on the augmented Lagrangian combined with the BFGS method. The evolution of the interior point algorithms (IPM) to solve large scale optimization problems led to the extensive formulation and solution of limit and shakedown analysis problems using these algorithms or related techniques (e.g. some representative publications [28–38]). In these works various applications of these procedures in many fields of solid and soil mechanics have been explicitly reported. For most recent applications one may look at [39].

Very few alternative approaches to finding the stationary point of an objective function exist in the literature for the evaluation of the shakedown load. One such approach is the eigen-mode method [40]. Also, a quite involved algorithm which is based on arc length techniques and is compared against the IPMs is presented in [36]. Another approach that uses internal variables, each of which corresponding to an inelastic mechanism, is presented in [41,42]. Using more physical arguments, the linear matching method (LMM) (originated in [43]) is a generalization of the elastic compensation method (e.g. [44,45]) and is based on matching a linear problem to a plasticity problem. It is an upper bound approach that generates a sequence of linear solutions, with spatially varying moduli, which converges to either the collapse load [46] or the shakedown load (e.g. [47,48]) of solid mechanics problems. Recent publications include applications to steel pipes [49] and concrete beams [50]. The method has also been carried over to shakedown problems in soil mechanics (e.g. [51,52]). It has also been extended beyond shakedown to provide an estimation of the ratchet boundary for a loading that can be decomposed into constant and time varying components ([53,54]). A recent update of the method has appeared in [55]. In [56] a numerical procedure was presented that uses the same loading assumptions.

A relatively simple direct method, called the Residual Stress Decomposition Method (RSDM), was presented recently [57,58] that may predict the long-term cyclic state of an elastic perfectly-plastic structure when