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A peridynamic material model for the analysis of dynamic crack propagation in orthotropic media

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Abstract

A new material model for the dynamic fracture analysis of anisotropic materials has been proposed within the framework of the bond-based peridynamic theory. This model enables predicting complex fracture phenomena such as spontaneous crack nucleation and crack branching, curving and arrest, a capability inherited from the bond-based peridynamic theory. An important feature of the model is that the bond properties, i.e. the stiffness constant and critical stretch, are continuous functions of bond orientation in the principal material axes. This facilitates fracture analysis of anisotropic materials with random orientations, such as polycrystalline microstructures. Elastic and fracture behaviour of the model has been verified through simulating uniaxial tension of a composite plate and fracture of a cortical bone compact tension specimen, and making quantitative comparisons to analytical and experimental data. To further demonstrate the capabilities of the proposed model, dynamic fracture of a polycrystalline microstructure (alumina ceramic) has been simulated. The influence of the grain boundary and grain interior fracture energies on the interacting and competing fracture modes of polycrystalline materials, i.e. intergranular and transgranular fracture, has been studied.

Keywords: Peridynamics; Anisotropy; Fracture; Bone; Ceramic

1. Introduction

Optimum design of most engineering structures requires accurate prediction of the fracture behaviour of materials. In the continuum mechanics theory, several techniques have been proposed and implemented in numerical programs to solve complex fracture problems. Much attention has been paid to isotropic materials, whose stiffness and strength properties are not direction-dependent. Prediction of fracture propagation in anisotropic media, such as composites, ceramics, rocks and bone, is also of great technological and clinical

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importance. However, this problem is more complicated because direction-dependence of stiffness and strength properties should be included in the formulation.

In order to predict fracture in anisotropic materials, researchers have extended numerical methods established for isotropic materials. Boone et al. [1] used the Finite Element Method (FEM) to simulate crack propagation in unidirectional (UD) fibre-reinforced composites and in rocks. Aliabadi and Sollero [2] proposed a 2D anisotropic material model suitable for implementation in a Boundary Element Method (BEM) code. The model was used to simulate crack propagation in UD fibre-reinforced composites [2] and in rocks [3,4]. Motamedi and Mohammadi [5] further developed the extended FEM (X-FEM) to simulate crack propagation in 2D anisotropic media, such as UD fibre-reinforced composite plies. In the approaches mentioned above, the maximum circumferential tensile stress theory is often used to estimate the crack propagation path. For a material with anisotropic fracture properties, the circumferential stress needs to be normalised with respect to the fracture properties; the crack grows in the direction where the normalised circumferential tensile stress is maximum. These methods also require quite complex laws to predict crack nucleation and branching, which are mostly developed for isotropic materials [6].

The peridynamic theory of continuum mechanics was proposed by Silling [7] to overcome some of the intrinsic limitations of the classical continuum mechanics theory when dealing with problems that contain discontinuous displacement fields, such as fracture problems. In this theory, each infinitesimal unit of the continuum, called particle, interacts with other particles located in its neighbourhood through forces, similar to the molecular dynamics theory [8,9]. The peridynamic theory is nonlocal since the interaction between particles extends beyond their immediate neighbourhood.

Anisotropic peridynamic models have been proposed by Xu et al. [10], Hu [11] and Oterkus and Madenci [12] to simulate crack propagation in UD fibre-reinforced composites. These models have been able to provide predictions of fibre/matrix fracture and delamination, which had good qualitative agreement with experimental observations. However, the models proposed in [10] and [12] require a uniform grid of particles and can only be used to model plies with a certain orientation. Hu et al. [11] proposed a method to remove these limitations, but it significantly adds to the computational cost for problems that use a non-uniform grid or a uniform grid with an arbitrary orientation of fibres. Another anisotropic peridynamic model was briefly described in Askari et al. [13] without presenting relevant equations, where the results of crack propagation in a polycrystalline microstructure were presented. In this paper, a new 2D anisotropic material model suitable for the bond-based peridynamic theory has been proposed. This model can be used with any discretisation of the domain. Several fracture problems are solved with this model and quantitative comparisons are made with analytical or experimental data.

This paper is organised as follows. The bond-based peridynamic theory is briefly reviewed in Section 2. The proposed material model is introduced in Section 3 followed by a description of the numerical implementation in Section 4. Using the proposed model, three example problems are solved in Section 5, where convergence of the solutions is discussed and the results are compared with experimental or analytical data. Some concluding remarks are presented in Section 6.

2. Bond-based peridynamic theory

In the bond-based peridynamic theory [7], the equation of motion of a particle at position $\mathbf{x}$ in the reference configuration is written as:

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t) = \int_{H_{\mathbf{x}}} \mathbf{f}(\mathbf{u}(\mathbf{x}',t) - \mathbf{u}(\mathbf{x},t), \mathbf{x}' - \mathbf{x}) dV_{\mathbf{x}'} + \mathbf{b}(\mathbf{x},t), \tag{1}$$

where ρ is the mass density, $\mathbf{u}$ is the displacement vector field, $\mathbf{f}$ is a pairwise force function, which is the force per volume squared that the particle $\mathbf{x}'$ exerts on the particle $\mathbf{x}$, and $\mathbf{b}$ is the body force vector field. $H_{\mathbf{x}}$ is a finite volume surrounding $\mathbf{x}$, referred to as the neighbourhood of $\mathbf{x}$. It is usually taken to be a sphere in 3D or a circle in 2D problems, centred at $\mathbf{x}$ (Fig. 1). Its radius, called the horizon, is denoted by δ. In order to ensure conservation of linear momentum and angular momentum, the following relations should hold:

$$\mathbf{f}(\boldsymbol{\eta}, \boldsymbol{\xi}) = -\mathbf{f}(-\boldsymbol{\eta}, -\boldsymbol{\xi}) \quad \forall \boldsymbol{\eta}, \boldsymbol{\xi}, \tag{2}$$