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A local wave tracking strategy for efficiently solving mid- and high-frequency Helmholtz problems

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Abstract

We propose a procedure for selecting basis function orientation to improve the efficiency of solution methodologies that employ local plane-wave approximations. The proposed adaptive approach consists of a local wave tracking strategy. Each plane-wave basis set within considered elements of the mesh partition is individually or collectively rotated to best align one function of the set with the local propagation direction of the field. Systematic determination of the direction of the field inside the computational domain is formulated as a minimization problem. As the resultant system is nonlinear with respect to the directions of propagation, the Newton method is employed with exact characterization of the Jacobian and Hessian. To illustrate the salient features and evaluate the performance of the proposed wave tracking approach, we present error estimates as well as numerical results obtained by incorporating the procedure into a prototypical plane-wave based approach, the least-squares method (LSM) developed by Monk and Wang (1999) [1]. The numerical results obtained for the case of a two-dimensional rigid scattering problem indicate that (a) convergence was achievable to a prescribed level of accuracy, even upon initial application of the tracking wave strategy outside the pre-asymptotic convergence region, and (b) the proposed approach reduced the size of the resulting system by up to two orders of magnitude, depending on the frequency range, with respect to the size of the standard LSM system.

Keywords: Direct scattering; Helmholtz equation; Adaptive direction; Plane-wave-based method; Least-squares method

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1. Introduction

Use of wave equations to model physical phenomena is well documented with wide-ranging applications in optics [2], seismology [3], radar [4], and ocean acoustics [5], along with many other fields in science and technology. While the ubiquitous finite element method has served as a foundation for the solution of this class of equations, issues that arise from the frequency dependence of the discretization, identified as a pollution effect [6,7], have remained a topic of active research for over a half a century. Many attempts have been made to overcome the difficulties related to this pollution effect. Relatively recently, approaches that employ plane-waves as basis functions for Helmholtz problems have demonstrated significant potential to numerically determine these solutions [1,8–21]. The oscillatory nature of plane-waves provides a natural setting to more efficiently model highly oscillatory fields. Nevertheless, fields that propagate with a high frequency remain difficult to compute, due to an increasing presence of numerical instabilities created upon refined discretization and/or the augmentation of the basis sets with additional plane-wave functions. These instabilities arise due to the numerical loss of linear independence of functions within the basis sets, as observed and demonstrated in [21,22].

In response to the above numerical challenges, we propose an alternative procedure that can extend the range of satisfactory convergence without significantly increasing the number of plane-waves and/or drastically refining the mesh. This can mitigate the nascent presence of near-linear dependencies that instigate numerical breakdown. The essence of the proposed approach is to maintain a low number of plane-waves, typically used to calculate fields propagating in the low frequency regime, to calculate fields at higher frequencies. This is accomplished by allowing the elemental basis sets to rotate so as to align a basis function in the set with the main direction of field propagation. In this manner, a more accurate approximation of the field is expected to that obtained by rigid and often arbitrarily predefined orientations of the basis sets. The proposed approach, which can be viewed as an adaptive-type strategy, is succinctly demonstrated by comparison of the analytical solution to the numerical one for a plane-wave propagating at an angle θ through a square wave-guide domain of length a [23]. In the domain, a high frequency propagation is considered ($ka = 500$) with the wavenumber represented by k. Using a basis set of four canonically oriented plane-waves per element, over 100% relative error was determined with the least-squares method (LSM) [1] for a step-size $h/a = 1/100$ for all propagation angles except those aligned with the predefined basis functions (0°, 90°, 180° and 270°), as depicted in Fig. 1. However, if the basis functions were allowed to rotate so that one function within an element aligns with the direction of propagation of the field, the “rotated” LSM delivered an error of 10^{-6}% with a much larger step size: $h/a = 1/2$, corresponding to a mesh partition of only two elements. Note that the same degree of rotation was applied to each of the four basis sets which is a logical option due to

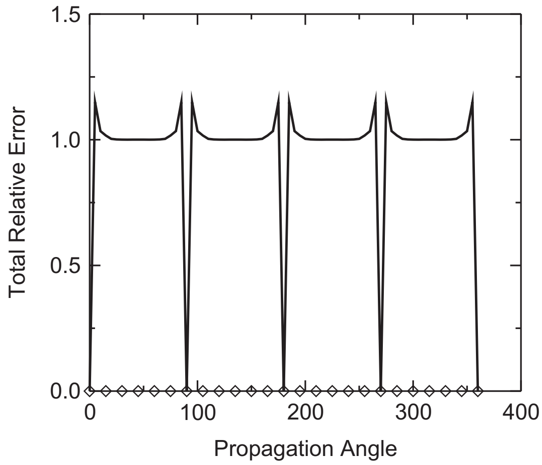

Fig. 1. Sensitivity of the relative error to the angle of propagation for $ka = 500$ and 4 plane-waves: LSM with $h/a = 1/100$ (line) and LSM-WT with $h/a = 1/2$ (diamond symbols).