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Comput. Methods Appl. Mech. Engrg. 276 (2014) 534–565

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Multimaterial topology optimization by volume constrained Allen–Cahn system and regularized projected steepest descent method

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Received 14 November 2013; received in revised form 19 March 2014; accepted 4 April 2014
Available online 18 April 2014

Abstract

A new computational algorithm is introduced in the present study to solve multimaterial topology optimization problems. It is based on the penalization of the objective functional by the multiphase volume constrained Ginzburg–Landau energy functional. The update procedure is based on the gradient flow of the objective functional by a fractional step projected steepest descent method. In the first step, the new design is found based on the projected steepest descent method to ensure the reduction in the objective functional, simultaneously satisfying the control constraints. In the second step, regularization step, an H^1 regularity of the solution is ensured while keeping the feasibility of solution with respect to the set of control constraints. The presented algorithm could be accounted as a constrained H^1 optimization algorithm, which, according to our knowledge, has not been reported to solve such kind of problems yet. The success and efficiency of the presented method are shown through several test problems. Numerical results show that the presented algorithm ends with a near 0–1 topology and its computational cost scales sub-linearly by the number of phases. For the sake of reader convenience and the ease of further extension, the MATLAB implementation of the presented algorithm is included in the appendix.

Keywords: Gradient projection; MATLAB code; Multiphase Allen–Cahn; Multiphase Cahn–Hilliard; Multiphase topology optimization; Sobolev gradient

1. Introduction

Topology optimization [1] aims at finding the distribution of a material in a fixed design domain such that an objective functional is minimized under certain constraints. In recent years there has been a growing

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http://dx.doi.org/10.1016/j.cma.2014.04.005

interest in solving topology optimization problems using multiple phases. The possibility of using multiple number of phases during design of engineering structures opens a new window toward the design of smart and advanced structures. It is particularly interesting for the optimal design of multifunctional structures, i.e., including a specific phase to manage a specific functionality. For instance, materials with high thermal conductivity usually have high thermal expansion coefficients and poor mechanical properties. On the other hand, high strength materials commonly have poor thermal conductivities. Therefore, using multiple number of phases appears to be a reasonable choice for the design of structures under thermomechanical loading. Moreover, multimaterial design strategy can reduce the total price or weight of structures, in contrast to the traditional single phase design paradigm. The later issue has been addressed in [2] by the mass constraint reformulation of multimaterial topology optimization problems. Solution of multimaterial topology optimization problems however includes several challenges. The lack of physically-based materials interpolation scheme [3] and efficient numerical method to solve the discretized form of the original problem is of these challenges. The later issue is our main goal in the present work.

The material interpolation here means computing the local value of desired physical properties based on the volume fractions of contributing phases. The solid isotropic material with penalization model (SIMP) [1], averaged Hashin–Shtrikman bounds [1] and homogenization methods [4,1] have been extensively used in the literature for (void-material) binary-phase topology optimization problems. The extension of these methods to manage arbitrary number of phases is not an easy job. For instance it is well known that the extended version of SIMP approach to three or more phases does not essentially follow the Hashin–Shtrikman bounds [1]. There are several attempts, e.g. [5,6], to develop general multimaterial interpolation schemes, there is however no progress toward developing a physically based material interpolation approach to mange arbitrary number of materials. Due to the simplicity and satisfactory results, the extended version of SIMP method has been extensively used in literature, e.g. [2,6–10], to perform the material interpolation. In practice, the material interpolation is an artificial construction in the context of the topology optimization, because a successful procedure results in a distinct phase within each computational element at the final solution [11]. Therefore, the interpolation model influences the optimization path in terms of the computational efficiency and the final design (note that the problem conconvexity leads to existence of different local optima). Appropriate schemes have to be developed for interpolation of the physical properties of multiphase materials. For instance, the existence of such interpolation schemes is a prerequisite for optimal design of multiphase functionally graded materials [12–14]. It is worth to mention that the sharp interface tracking approaches, like the level set method [15], can eliminate the need of material interpolation schemes. It is important to note that this issue is only correct if inter-phase interfaces are actually tracked explicitly (i.e. one does not have a computational cell including more than one phase), which is not the case of matter in many level set based methods.

Using the homogenization method, the design of three-phase composites with extremal thermal expansion, piezoelectricity and bulk modulus have been considered in [16–19] respectively (cf. [20]). To avoid the topological instability (cf. [21]) and posing control on the minimum length scale, the low-pass sensitivity filtering [1] has been included in these works. Using combined multiphase SIMP approach and sensitivity filtering, optimal design of piezoelectric actuators has been considered in [9]. In these works, authors however did not attend to numerical solution of resulted systems of optimality conditions and passed it to a general purpose optimization black-box. In [22], the optimization of the position of fuel assemblies in a nuclear reactor core has been formulated as a four-phase topology optimization problem based on the homogenization theory. Then, it has been solved numerically using a simple constrained gradient descent approach. Using the block coordinate descent approach, the classical optimality criteria approach [23,1] has been extended in [10] to solve multimaterial topology optimization problems.

The level set and the variational level set approaches have been adapted in [22,24,11,25–27] to solve multimaterial topology optimization problems. The piecewise-constant variational level set method has been used in [28,29] to improve the computational cost of multiple level set based approaches. The dependency of the final topology to the initial design appears to be the main shortcoming of the level-set based methods.

Optimization of laminated composite shell structures using multimaterial topology optimization strategy, called discrete material optimization (DMO), has been introduced in [30,31]. In this method, every virtual mate-