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An interior point method for isogeometric contact

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Abstract

The interior point method is applied to frictionless contact mechanics problems and is shown to be a viable alternative to the augmented Lagrangian approach. The method is derived from a mixed formulation which induces a contact discretization scheme in the spirit of the mortar method and naturally delivers slack variables that help constrain the solution to the feasible region. The derivation of the algorithm as well as its robustness benefits from the contact interface description that is induced by NURBS-based isogeometric volume discretizations. Various interior point algorithms are discussed, including a primal–dual approach that satisfies the unilateral contact constraints exactly, in addition to two primal approaches that retain an arbitrary barrier parameter. The developed algorithms can easily be pursued starting from an augmented Lagrangian implementation. Numerical investigations on benchmark problems demonstrate the efficiency and the robustness of the framework, but also highlight current limitations that suggest paths for future research. Overall, the results indicate that the interior point method can challenge the augmented Lagrangian method in contact mechanics, even displaying potential for higher efficiency and robustness.

Keywords: Contact; Interior point method; Mortar discretization; Isogeometric analysis

1. Introduction

Mathematically, contact poses an optimization problem with inequality constraints. It is therefore no surprise that the augmented Lagrangian (AL) method [1] has gradually established itself as the most favorable method for solving contact mechanics problems with simultaneous efficiency and robustness, since it is one of the most favored methods for nonlinear optimization problems as well. However, recent advances suggest the interior point (IP) method to be a viable alternative — see [2] for an extensive review. In its simplest form, the IP method is the classical barrier method [3,4]. The form of the barrier function can be physically

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motivated by microscopic contact mechanisms [5,6]. From an optimization point of view, on the other hand, the barrier is classically logarithmic. Independent of the particular choice, a straightforward application of any barrier function faces challenges, for instance due to the need to prevent the penetration of the surfaces in order to avoid an unphysical interface pressure. The choices made in addressing these challenges directly impact the efficiency and the robustness of the IP method. Indeed, early comparison studies indicated that the AL approach is more robust [7]. In this work, it will be demonstrated that the IP method can challenge the AL approach, even displaying potential for higher efficiency and robustness.

In comparison to penalty, Lagrange multiplier and AL methods, there is only a limited number of studies concentrating on the application of the IP method in its modern form to contact mechanics. The work [7] already considered Coulomb friction and additionally introduced slack variables that are crucial to effectively constraining the solution to the feasible region — see also [8] for similar studies without friction. A solution scheme that is reminiscent of Uzawa staggering for the multiplier updates in the AL approach has been considered in [9] for frictionless contact, also using slack variables. Both of the works are in a two-dimensional setting, the former limited to linear elasticity and a single deformable body while the latter considers hyperelasticity with two deformable bodies and non-matching meshes based on a node-to-segment strategy for formulating contact. A hybrid method based on a combination of the AL and the IP methods was proposed in [10] for the linearly elastic frictionless contact of two bodies with matching meshes in a node-to-node setting. Kučera et al. [11] considers a linearly elastic three-dimensional body in contact with a rigid obstacle, with Coulomb as well as Tresca friction. One can also identify applications in the optimization literature, for instance [12,13]. While all of these works are based on the finite element method, the boundary element method can also be an efficient choice for large scale linear elasticity problems with contact [14].

There are two aspects to investigating an IP method for contact problems. The optimization aspect is still an active research area. The algorithm employed in this work is an established framework yet leaves room for improvement. For all aspects of the IP method from an optimization perspective, from the terminology to the methods chosen, [15] will be made use of extensively and without further reference unless needed for clarity. See also [16] for an overview. The second aspect concerns the discretization of the problem, both that of the bodies as well as the contact variables. To some extent, the optimization algorithm can be varied for a given discretization method, although the particular method will strongly affect the overall robustness of the framework. Presently, the aim is to develop and test a strategy that is suitable for three-dimensional, frictionless finite deformation contact problems with non-matching meshes. The mortar method has established itself as the most robust contact discretization approach in this context and will be made use of in this work — detailed references will be provided in upcoming sections. The discretization of the volume interacts naturally with the contact discretization. Here, NURBS-based isogeometric analysis will be employed, initiated in [17] and reviewed in [18]. Recent applications include porous media modeling [19], boundary element analysis [20], fluid mechanics [21] and shells [22] with optimization [23]. Isogeometric contact approaches, where a smooth surface finite element discretization is naturally induced by the volume discretization, were initiated in [24,25]. Various advantages reported in recent works will be highlighted. Nevertheless, to the best knowledge of the authors', the present work also constitutes a first attempt towards combining IP methods with the mortar idea. Therefore, although not pursued in this work, the use of mortar formulations developed on the basis of Lagrange discretizations would be equally interesting.

The work is organized as follows. In Section 2, the basic barrier regularization idea behind the IP method is reviewed. The mixed formulation basis of the regularized formulation is presented, where slack variables naturally appear and provide a means for avoiding an unphysical interface pressure. The mixed formulation additionally induces a contact discretization scheme in the spirit of the mortar method, as outlined in Section 3. In particular, a lumped formulation that is convenient for numerical implementation will be derived through a series of simplifying assumptions. A primal–dual strategy is then discussed in Section 4 where the aim is to reduce the barrier parameter towards zero. This is realized within a two-stage predictor–corrector scheme embedded in an iteration of the Newton–Raphson method for solving the nonlinear problem. Alternative primal formulations will additionally be introduced as simplifications of the primal–dual strategy. Numerical investigations will be provided in Section 5 where two-body frictionless elastic contact problems will be discussed and comparisons with the AL method will be carried out.