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Comput. Methods Appl. Mech. Engrg. 276 (2014) 627–658

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

A new perspective on the solution of uncertainty quantification and reliability analysis of large-scale problems

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Received 22 November 2013; received in revised form 17 March 2014; accepted 23 March 2014
Available online 26 April 2014

Abstract

This work revisits the computational performance of non-intrusive Monte Carlo versus intrusive Galerkin methods of large-scale stochastic systems in the framework of high performance computing environments. The purpose of this work is to perform an assessment of the range of the relative superiority of these approaches with regard to a variety of stochastic parameters. In both approaches, the solution of the resulting algebraic equations is performed with a combination of primal and dual domain decomposition methods implementing specifically tailored preconditioners. The solution of repeated simulations of the Monte Carlo method is accelerated with an A-orthogonalization procedure aiming at reducing the iterations of subsequent simulations, while the solution of the augmented equations of the stochastic Galerkin method is enhanced with preconditioners which combine the block diagonal features of the resulting matrices as well as the sparsity pattern of the off block-diagonal terms. Numerical results are presented, demonstrating the efficiency of the proposed implementations on a large-scale 3D problem with different stochastic characteristics and useful conclusions are derived regarding the ranges of stochastic parameters in which non-intrusive solvers have a superior performance compared to intrusive ones and vice versa.
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Keywords: Spectral stochastic finite element method; Monte Carlo simulation; Polynomial chaos; Karhunen–Loève decomposition; Primal–dual domain decomposition; FETI method

1. Introduction

The most straightforward technique of solving stochastic partial differential equations (PDE) are the widely applicable non-intrusive Monte Carlo (MC) methods. They can handle any type of problems (linear, nonlinear,

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http://dx.doi.org/10.1016/j.cma.2014.03.009
0045-7825/© 2014 Published by Elsevier B.V.

dynamic) as well as any kind of uncertainty in the load or in the system properties and they can be implemented in a non-intrusive manner in the framework of existing deterministic solvers. In particular, when dealing with deterministic external loading, Monte Carlo methods feature the solution of successive linear systems with multiple left-hand sides, since only the coefficient matrix **K** changes in every simulation. Due to the fact that the solution process has to start from the beginning, a new stiffness matrix needs to be formed at each simulation. Thus, the repeated solutions of the system of equations for each newly formed solution becomes a major computational task that hinders the stochastic assessment of large-scale problems with MC methods. Such solution can be performed either with a standard direct method based on Cholesky factorization or with iterative methods. The solution with a direct method has two major drawbacks: poor performance in 3D large scale problems and in parallel and distributed computing environments, as well as incapability of exploiting the near-by nature of the successive simulations.

In order to alleviate the incapability of direct schemes to exploit the proximity of the resulting systems of equations, iterative solvers have been proposed which are customized to the particular properties of the equilibrium equations arising in the context of MC methods. Such iterative solvers have been presented for sequential [1–5] as well as for parallel computing environments [6–11]. The resulting near-by problems can be effectively solved using the preconditioned conjugate gradient (PCG) algorithm equipped with a preconditioner following the rationale of incomplete Cholesky preconditioning. This solution procedure consists of utilizing the deterministic $\mathbf{K}_0$ stiffness matrix as its preconditioner throughout the entire simulation process for the solution of the near-by problems. The repeated solutions required for the preconditioning step of the MC-PCG-Skyline algorithm can be treated as problems with multiple right-hand sides, since the entries in the residual vector are updated at each PCG iteration of each MC simulation. A direct solver for this procedure is proposed in [12] and the dual decomposition FETI method is proposed in [11].

On the other hand, recently proposed approaches, such as stochastic collocation and Galerkin methods, are intrusive and are using tensor product spaces for the spatial and stochastic discretizations. In the case where the uncertain input parameters are modeled via the Karhunen–Loève (KL) expansion and the system response is projected on a polynomial chaos (PC) basis, the method is called spectral stochastic finite element method (SSFEM). SSFEM approach applies a Galerkin minimization in order to transform a stochastic PDE into a coupled set of deterministic PDEs. Thus, the solution of stochastic problems using the SSFEM approach has to be performed on augmented linear equation systems which can be up to orders of magnitude larger than the corresponding deterministic ones [13–21].

For large-scale problems the solution of such augmented algebraic systems can become quite challenging due to the increased memory and computational resources required. Solution techniques for addressing these problems are based on adaptive methods [22,23] and on iterative solvers like the block Gauss–Jacobi [8,24–26] and the PCG [2,5,8,18,27–32]. The variant proposed in [32] is an extension of the domain decomposition FETI-DP method to SSFEM problems where the Lagrange multipliers are enhanced in order to take into account the interaction forces occurring at both the deterministic and stochastic parts. Although such an implementation might prove efficient when compared to the aforementioned iterative solvers, its performance can be degraded when dealing with large KL and PC expansion orders. The two-level variant proposed in [29] proves to be quite efficient since it exploits the block-sparsity structure of the augmented stiffness matrix while utilizing a domain decomposition solver, optimized for multiple right-hand sides.

The present work revisits the computational performance of non-intrusive Monte Carlo versus intrusive Galerkin methods for large-scale stochastic systems in the framework of high performance computing environments. The purpose of this work is to perform an assessment of the range of the relative superiority of these approaches with regard to a variety of stochastic parameters. In both approaches, the solution of the resulting algebraic equations is performed with a combination of primal and dual domain decomposition methods implementing specifically tailored preconditioners. The solution of repeated simulations of the Monte Carlo method is accelerated with an A-orthogonalization procedure aiming at reducing the iterations of subsequent simulations, while the solution of the augmented equations of the stochastic Galerkin method is enhanced with preconditioners which combine the block diagonal features of the resulting matrices as well as the sparsity pattern of the off block-diagonal terms. Numerical results are presented demonstrating the efficiency of the proposed implementations on a variety of problems with different stochastic characteristics and useful

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