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Comput. Methods Appl. Mech. Engrg. 275 (2014) 39–54

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

In-plane mesh regularization for node-based shape optimization problems

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Received 10 December 2012; received in revised form 1 February 2014; accepted 22 February 2014
Available online 5 March 2014

Abstract

In this contribution a global and linear method for controlling the surface mesh quality during node-based shape optimization is presented. In this method, an artificial stress is applied on the surface mesh and the global linear system of equations is solved for the equilibrium state by finite elements. The applied stress adapts each element towards a desired predefined template geometry and at the end a globally smooth mesh is achieved. In this way both the shape and the size of each element is controlled. Some examples, as well as possible fields of application are shown. At the end, the strength of the method in shape optimization of computational fluid dynamics problems is described.

Keywords: Aerodynamic shape optimization; Node-based; CAD-free; Mesh smoothing; In-plane regularization

1. Introduction

Optimal shape design receives great attention in aerospace, marine and automotive industry where the aim is to minimize a functional that describes mechanical characteristics of the design. A key element in this process is the definition of the parametrization of the shape. Inspired from engineering design, Computer Aided Geometric Design (CAGD) methods have been commonly used to represent the shape for both structure and fluid problems. In this instance, the design parameters of the CAD are the design variables of the optimization problem. The limited number of design variables allows efficient computation of direct sensitivities. In contrary, the low number of design variables restricts the design space not being able to produce all the shape modes needed to capture the optimum.

Adjoint methods provide sensitivity values on every discretization element (cell, point, etc.) almost by the same cost as the primal problem [1,2]. This motivates the use of enriched design spaces with larger number of

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http://dx.doi.org/10.1016/j.cma.2014.02.013
0045-7825/

design variables. In contrast to CAD methods, the node-based shape optimization approach regards the design space to be as large as possible by considering directly the node positions as design variables. Hence, the discretization of the design surface is also used to describe the geometry and provides the vertex coordinates as design variables for the optimization problem.

This method suffers from mesh dependent results and non-smooth derivatives which almost always lead to unphysical and jagged shapes [3]. For this reason various projection methods are suggested which smoothen the variation of the shape in the "out of plane" direction [4–7]. However, updating the discrete surface only in the normal direction without regarding any changes in the tangent to the surface direction can lead to high distortion of the surface elements [5,8,9]. As a result, an "in-plane" regularization step is also required to ensure the quality of the surface discretization during optimization.

Generally, in the field of mesh quality control there are two main classes of problems: the mesh quality improvement and the mesh motion problems. In mesh smoothing, the goal is to improve the quality of a 2D or 3D mesh [10–13] whereas in mesh motion problems the resulting 2D or 3D mesh is seeked after moving a 1D or a 2D boundary respectively [14–16]. In both fields the ideas are similar. As the mesh motion solvers advance, bigger boundary updates can take place. These larger modifications of the design surface give rise to the mesh distortion problem of the boundary. For this reason, in-plane regularization methods are required which deal with retaining the surface mesh quality during the evolution of the shape [5,8].

Within this contribution, a global method which regularizes the surface mesh to a desired condition is presented. In this method, an artificial stress field is applied on the surface or on the volume mesh and a global linear system for the equilibrium is solved. The applied stress adapts the shape of each element towards a desired predefined template geometry and at the end a globally smooth mesh is achieved. In this way, both shape and size of each element is controlled. The method can be applied on both structured and unstructured grids since there is no additional assumption on the mesh topology.

The remainder of this article is organized as follows: Section 2 introduces the shape optimization problem and the in-plane regularization term of the augmented problem. In Section 3, a short overview is given on the methods used so far for mesh smoothing and mesh quality control. The proposed in-plane regularization method is discussed in Section 4 and in the next section, the role of the fundamental components of the method is demonstrated. Finally, numerical results of minimization of power loss in a ducted flow and conclusions are presented in sections 6 and 7, respectively.

2. The optimization problem

In general, in node-based shape optimization the shape is described only by the discretization and no other geometrical link is established. Therefore, the coordinates of the surface nodes are considered to be the design variables of the optimization problem, since changing the position of the internal nodes will not alter the shape. More precisely, the position of the surface point can be decomposed into two components, normal and tangential to the surface, as follows:

$$\boldsymbol{x}_l = x_{l,n} \cdot \boldsymbol{n} + x_{l,t} \cdot \boldsymbol{t} = s_l \cdot \boldsymbol{n} + r_l \cdot \boldsymbol{t}, \quad l = 1, \ldots, n_s. \tag{1}$$

n_s is the number of surface nodes and $\boldsymbol{n}$ and $\boldsymbol{t}$ are the unit vectors normal and tangential to the surface at node l.

Neglecting the discretization error and the finite step size of the optimization, small variations of $x_{l,n}$ in the "out of plane" direction will cause a change in the shape, while small variation of $x_{l,t}$ in the "in-plane" direction will only alter the discretization. In other words, $x_{l,n}$ is the shape relevant component s_l, while $x_{l,t}$ is the mesh relevant component r_l. For this reason only the normal component s_l is regarded as design variable and consequently in each optimization step only the out of plane direction is updated. However, during this process the quality of elements could deteriorate and severely distorted elements might appear. In extreme cases, the elements become degenerate and further progress of analysis is restricted. Hence, a correction in the "in-plane" direction is required.

For this reason, the optimization problem with response function f, constraints (g_j, h_k) and variable bounds $(s_l^{lower}, s_l^{upper})$ for each surface node l is modified as follows: