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Detection of multiple flaws in piezoelectric structures using XFEM and level sets

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Abstract

An iterative procedure to solve the inverse problem of detecting multiple voids in piezoelectric structure is proposed. In each iteration the forward problem is solved for various void configurations, and at each iteration, the mechanical and electrical responses of a piezoelectric structure is minimized at known specific points along the boundary to match the measured data. The Extended Finite Element method (XFEM) is employed for determining the responses as it allows the use of a fixed mesh for varying void geometries. The numerical method based on combination of classical shape derivative and of the level-set method for front propagation used in structural optimization is utilized to minimize the cost function. The results obtained show that this method is effectively able to determine the number of voids in a piezoelectric structure and its corresponding locations and shapes.

Keywords: Inverse problem; XFEM; Level set method; Piezoelectric ceramics

1. Introduction

Piezoelectric materials deform when subjected to an electric field and produce an electric field when stressed. This intrinsic coupling behavior of piezoelectric materials has attracted wide applications in electro-mechanical and electronic devices such as electro-mechanical actuators, sensors and transducers. In general, some defects like voids or cracks may be produced in piezoelectric materials during the manufacturing process. When they are subjected to mechanical and electrical loads, stress concentrations due to these defects may lead to critical crack growth and subsequent mechanical failure or dielectric breakdown. Many researchers have studied the

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behavior of these materials in the presence of defects; fundamentals of piezoelectric fracture mechanics can be found in [1–4]. A short overview and a critical discussion about the present state in the field of piezoelectric fracture mechanics is given in [5]. Application of piezoelectric fracture mechanics to realistic crack configurations and loading situations requires an effective numerical method like the Finite Element Method (FEM), meshfree methods, phantom node or numerical manifold method [6] or DDA [7]. A review on the state of art of applying FEM to analyse cracks in 2D and 3D piezoelectric structures is given in [8].

In Rus et al. [9] a series of studies on damage detection in piezoelectric materials is presented, in which the inverse problem is solved iteratively using the FEM and Boundary Element Method (BEM). In this work, the cost functional is minimized using a Genetic Algorithm (GA). An enhanced iterative scheme for the precise reconstruction of piezoelectric material parameters from electric impedance and mechanical displacement measurement is presented in [10]. In works related to crack or void identification a remeshing of the finite element domain is required in each iteration of the optimization scheme to solve the inverse problem. Mesh free methods [11–16] do not require remeshing and they have been used to solve inverse problems [17] iteratively. On the other hand, the Extended Finite Element method (XFEM) has been utilized to solve the forward problem in each iteration in [18,19]. In XFEM [20,21], the displacement field is enriched near the crack face by incorporating both discontinuous fields and near tip asymptotic fields through a partition of unity method. XFEM exploits the partition of unity property of Finite Elements identified by Melenk and Babuska [22], which allows local enrichment functions to be easily incorporated into a Finite Element approximation. In XFEM, implicit level set functions are used to model cracks [23], holes and material interfaces (inclusions) [24]. Some improvements in XFEM are proposed in [25] so as to obtain improved performances. Application of XFEM to the analysis of fracture in piezoelectric materials is presented in Bechet et al. [26], where new crack tip enrichment functions suitable for cracks in piezoelectric structures are proposed. An Extended Finite Element formulation for dynamic fracture of piezoelectric materials is developed in [27]. The inverse problem of detecting cracks and voids in 2D piezoelectric structures using XFEM is presented in [28]. The optimization schemes utilized commonly in solving the inverse problem of damage detection are Genetic Algorithm [18] and global search methods [28]. The number of iterations in these methods increases in proportion to the number of parameters used to define the flaws. Because of this limitation, most previous studies were restricted to detecting only one single void or crack of simple geometry. For example, in [28], the void is explicitly defined using five parameters. These parameters are determined such that the objective function is minimized using Multilevel Coordinate Search [29] as the optimization algorithm. The algorithm proposed can determine the location and equivalent elliptical shape of only one single void in a piezoelectric structure.

A new numerical method based on the combination of the classical shape derivative and of the level-set method for front propagation in the context of structural optimization is proposed in [30]. XFEM based level set schemes for structural optimization are presented in [31].

The aim of this paper is to propose a strategy to detect multiple voids in 2D piezoelectric structures by combining shape derivative and level sets as employed in structural optimization problems. XFEM, in addition to independence of background mesh to flaw configuration, utilizes implicit level set functions for defining flaws. Thereby, it becomes a natural choice for solving the forward problem in each iteration for different flaw configurations.

The outline of the paper is as follows. Sections 2 and 3 are about piezoelectric governing equations and the piezoelectric Extended Finite Elements formulation respectively. Section 4 comprises details on combining shape derivative and level set method to minimize the objective function and thereby detecting the location of voids. Section 5 shows numerical examples to prove the ability of this method in solving the intended inverse problem iteratively.

2. Basic piezoelectric relations

The constitutive equations of a linear piezoelectric material are,

$$\sigma_{ij} = C^E_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \tag{1}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \kappa^{\varepsilon}_{ik}E_k. \tag{2}$$