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Rotationally invariant distortion resistant finite-elements

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Abstract

The predictive capability of conventional iso-parametric finite-elements deteriorates with mesh distortion. In the case of geometrically non-linear analysis, changes in geometry causing severe distortion can result in negative Jacobian mapping between the local and global systems resulting in numerical breakdown. This paper presents a finite-element formulation that is resistant to irregular mesh geometries and large element distortions whilst remaining invariant to rigid body motion. The predictive capabilities of the family of finite-elements are demonstrated using a series of geometrically non-linear analyses including an elastic cantilever beam and an elasto-plastic double notched specimen.

Keywords: Finite-elements; Mesh distortion; Geometric non-linearity; Elasto-plasticity; Shape functions

1. Introduction

Three-dimensional iso-parametric hexahedral elements generally perform best when in the form of right regular cubes. However, in practice, to reproduce irregular geometries this is not possible. Further, it is well known that the accuracy of the finite-element solution deteriorates as elements become distorted. Distortion typically occurs when meshing complex curved geometries or when simulating large deformation processes like forging or extrusion. Distorted elements can lead to not just inaccurate results but, in extreme cases, breakdown of the numerical algorithm through a negative Jacobian mapping between the local and global systems. At this point, one is forced down the computationally expensive task of re-meshing and transferring the state variables and internal forces to the new discretisation. The problem of mesh distortion sensitivity has been apparent since the seventies (for example, see [5,6] amongst others), however a complete solution to the problem has yet to emerge.

Although there have been several alternative approaches to overcoming the distortion sensitivity of finite-elements, such as the smoothed finite-element method [9] and meshless approaches (see Ullah [22] for

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an overview), the tremendous popularity of standard finite-elements justifies continued research within this classical framework.

Perhaps the most promising method to produce distortion-*immune* finite elements was introduced by Rajendran and his co-researchers. Rajendran and Liew [17] presented an unsymmetric 8-noded quadrilateral element, reporting that this element could exhibit immunity to any kind of mesh distortion under a quadratic displacement field. They named their new element the US-QUAD8 (unsymmetric quadrilateral element with eight nodes). However, a disadvantage with this formulation is it produces an unsymmetric stiffness matrix requiring an unsymmetric global solver. However, these elements were able to resist mesh distortion and were capable of producing accurate results despite heavy mesh distortion (under certain displacement fields). The element proposed by Ranjendran and Liew is only distortion-immune when the underlying basis of the trial functions can exactly capture the displacement field. In this paper we will instead use the term 'distortion-resistant' rather then distortion immune.

Ooi et al. [12] extended the idea of these distortion-resistant unsymmetric elements to three dimensions, proposing an unysmmetric 20-noded hexahedral element (US-HEXA20). Liew, et al. [8] introduced a two-dimensional, 6-noded triangular element based on the same underlying formulation. Prathap, et al. [15] investigated the approach using the best-fit paradigm. They observed that when an element was distorted, the isoparametric shape functions (used as the test functions) helped satisfy continuity across the element's edge. Using metric shape functions for the trial basis ensured completeness across the element, allowing exact reproduction of the appropriate order displacement field. This observation explained why the unsymmetric formulation give excellent results for distorted meshes. Another observation made in this paper was the lack of the determinant of the Jacobian matrix in the stiffness integral. This allowed accurate calculations of the stiffness integral, even when the determinant of the Jacobian went negative as a consequence of mesh distortion. However, the Jacobian matrix does feature in the unsymmetric formulation, albeit in an alternative guise.

Rajendran et al. [18] further investigated the mesh distortion immunity for the QUAD8 elements using constant, linear and quadratic strain field patch tests. In 2008, Ooi et al. [14] highlighted two defects associated with the US-QUAD8. These were its rotational frame dependence and interpolation failure under certain conditions. The remedy to its rotational frame dependence proposed in that paper was given by as rotating the local coordinate system to coincide with one of the element's edges. Although this did produce a formulation invariant to rigid body rotations, care must be taken to select an appropriate edge. Furthermore, the extension to three-dimensions is not clear. Interpolation failure of the metric shape functions is easy to identify, occurring when the functions do not sum to unity. In cases when this occurred, a random small transformation of the coordinate system was applied, the element constructed and the stiffness matrix was transformed back to its original configuration. Rajendran's 2010 paper [16] provided a comprehensive study into the 8-noded quadrilateral unsymmetric element formulation. The work extended Prathap et al.'s [15] studies and investigated how the absence of the Jacobian matrix from the stiffness integral may help reduce inaccuracy due to mesh distortion. The usymmetric elements of Rajendran and co-workers have also recently been applied to the analysis of finite deformation elastic problems using a total-Lagrangian framework [13].

An alternative to these unsymmetric formulations is the use of hybrid stress-function (HSF) elements (see, for example [2] and the references contained within). HSF elements are based on the principle of minimum complementary energy and their basis functions are obtained from analytical solutions of the Airy stress function. They are able to withstand high mesh distortion and are rotationally invariant. However, a number of points should be noted about these HSF elements: (i) they require significantly higher Gauss-Lagrange quadrature compared to conventional finite-elements and (ii) careful selection of an appropriate number of trial functions is essential to ensure that spurious energy modes do not appear in the element stiffness matrix (this selection process is not yet fully understood [2]). The greatest restriction of the HSF formulation is inherent in the use of Airy stress function solutions, limiting the elements to elastic analysis.

A recent paper by Cen et al. [3] proposed an element that combined the unsymmetric approach of Rajendran and co-workers [16,17] with Cen et al.'s [2] HSF formulation. The element overcame the rotational invariance and interpolation failure problems of the unsymmetric formulation by replacing the metric shape function with basis functions from the Airy stress function solution. However, as with the HSF approach, this restricts the element to elastic analysis.