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Dynamic analysis of a poroelastic layered half-space using continued-fraction absorbing boundary conditions

Jin Ho Lee [a,1], Jae Kwan Kim [b], John L. Tassoulas [c,*]

[a] BK21 Safe and Sustainable Infrastructure Research, Department of Civil and Environmental Engineering, Seoul National University, Seoul 151-744, Republic of Korea
[b] Department of Civil and Environmental Engineering, Seoul National University, Seoul 151-744, Republic of Korea
[c] Department of Civil, Architectural and Environmental Engineering, The University of Texas at Austin, Austin, TX 78712-0273, USA

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ABSTRACT

A methodology for dynamic analysis of a poroelastic layered half-space is proposed. A poroelastic layered half-space is discretized with regular thin layers to a certain depth. Below the layers, continued-fraction absorbing boundary conditions (CFABCs) which are accurate and effective in modeling wave propagation in various unbounded domains are applied in order to represent the infinite extent of the half-space. With the representation, a spectral decomposition which is an effective approach in the solution of wave-propagation problems is utilized. Green functions of a poroelastic layered half-space are obtained accurately with the proposed numerical model. The Green functions can be applied to various dynamic problems in the half-space including foundation dynamics and soil–structure interaction problems.

1. Introduction

Wave propagation in poroelastic layered media is relevant in seismology, civil engineering, geotechnical engineering, and petroleum engineering. Typical examples are analysis of dynamic soil–structure interaction for structures built in poroelastic layered ground and subsurface imaging to understand deep geologic structures and explore hydrocarbon deposits. In order to examine exact dynamic behaviors of the media, analytical solutions and numerical methods have been developed.

Most studies on poroelastic media are based on Biot's theory [1,2]. It was shown in early research that there are three kinds of waves in the medium, i.e. $P1$, $P2$, and S waves. The effects of various boundary or interface conditions on wave propagations in the medium were studied in a series of papers by Deresiewicz and his co-workers [3–12]. Based on Biot's theory, fundamental solutions in a poroelastic full space have been obtained for various wave-propagation problems. Fundamental solutions for a point force in the solid component were obtained by Burridge and Vargas [13] and Philippacopoulos [14], those for a point force in both the solid and the fluid components by Norris [15] and Kaynia and Banerjee [16], 2.5-D solutions by Lu et al. [17], transient solutions for 2-D and 3-D continua by Chen [18,19], and solutions in a transversely isotropic poroelastic material by Kazi-Aoual et al. [20]. A system of boundary integral equations and fundamental solutions were derived in the Laplace transformed domain and an analogy between poroelastodynamics and thermoelasticity was drawn by Manolis and Beskos [21]. Bonnet [22] also obtained fundamental solutions from the analogy. A direct boundary element approach and fundamental solutions in the time domain were developed by Wiebe and Antes [23].

Solutions in a poroelastic half-space have been obtained for various wave-propagation problems. Half-space solutions for an impulsive line load and a circular uniform surface load on the solid constituent were derived by Paul [24,25], solutions for tractions in the solid and fluid by Halpern and Christiano [26], solutions for buried loads in a homogeneous poroelastic half-space by Senjuntichai and Rajapakse [27], solutions in a layered half-space by Rajapakse and Senjuntichai [28] and Degrande et al. [29], and solutions in a 3-D poroelastic half-space by Jin and Liu [30]. Philippacopoulos [31] solved Lamb's problem for poroelastic half-space.

Numerical methods as well as analytical solutions for an infinite poroelastic domain have been developed. Zienkiewicz et al. [32] and Lewis and Schrefler [33] presented finite element formulations for a poroelastic medium. Special treatments must be considered in order to represent an infinite medium within the framework

* Corresponding author. Tel.: +1 512 471 4553; fax: +1 512 471 7259.
E-mail address: yannis@mail.utexas.edu (J.L. Tassoulas).
[1] Current Address: New Transportation Systems Research Center, Korea Railroad Research Institute, Uiwang-City, Gyeonggi-Do 437-757, Republic of Korea.

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of the finite element method. Degrande and De Roeck [34] presented an absorbing boundary condition which corresponds to a viscous boundary condition for an elastic medium. An infinite element was formulated by Khalili et al. [35]. Boundary element formulations which are suitable for an infinite medium have been presented as well. Boundary element approaches were obtained in the Laplace transformed domain by Manolis and Beskos [21], in the frequency domain by Cheng et al. [36] and Domínguez [37] and in the time domain by Wiebe and Antes [23] and Chen and Dargush [38]. Rajapakse and Senjuntichai [39] developed an indirect boundary integral equation method for quasi-static, time-harmonic and transient boundary value problems. A comprehensive review on poroelastodynamics can be found in the paper by Schanz [40].

Since wave propagation in elastic or poroelastic *layered* media is relevant in many applications, as mentioned above, many researchers have studied the topic and developed various methods. Thomson [41] and Haskell [42] presented formulations for wave propagation in elastic layered media based on transfer matrices. Kausel and Roësset [43] presented an alternative method using stiffness matrices. The stiffness-matrix approach was extended to poroelastic media by Rajapakse and Senjuntichai [28] and Degrande et al. [29]. Since transcendental functions appear in the stiffness matrices for finite layers, closed-form solutions are possible only for problems of simple geometries by contour integration. General solutions for arbitrary layered media can be obtained by numerical integration over a wavenumber domain. The numerical method usually involves spatial aliasing or numerical errors associated with either truncation or the large and rapid oscillations of the kernels.

If the transcendental functions in the stiffness matrices are represented by means of Taylor series expansions, the numerical integration can be avoided and the integral transform can be evaluated exactly. Subdividing a finite layer into several thin layers of thicknesses small compared to the minimum wavelength of interest and adopting simple interpolation functions in the direction of layering, the transcendental functions can be avoided and discrete layer stiffness matrices can be obtained [43]. It can be shown easily that the subdivision is equivalent to a representation of the transcendental functions in Taylor series expansions [44]. The discrete stiffness matrices can be written as quadratic functions of a horizontal wavenumber. From the discrete stiffness matrices, a quadratic eigenvalue problem can be formulated and the spectral decomposition can be applied in order to obtain wave modes in layered media. Using the wave modes, the integral transforms can be evaluated exactly [45,46]. The method based on the spectral decomposition of the discrete stiffness matrices is referred to as a 'thin-layer method'. Since the method leads to appropriate and effective numerical models, it has been applied to the analysis of various layered systems [43,45–60].

But the method based on the spectral decompositions requires special care when applied to problems in a layered *half-space*. The half-space is usually replaced by a layered stratum on rigid bedrock at sufficient depth. However, the fixed boundary condition can lead to unsatisfactory results, especially at low frequencies, if the depth of the stratum is not sufficient. To overcome this difficulty, second-order paraxial approximations of exact half-space conditions [56,57] and continued-fraction absorbing boundary conditions (CFABCs) [61,62] have been implemented in order to enhance the method based on the spectral decomposition [56–60]. Since the CFABCs are accurate and effective in modeling wave propagation in various unbounded domains [61–68], it is desirable to apply the CFABCs in the framework of the spectral decomposition.

In this study, CFABCs for *poroelastic* media are combined with the discrete stiffness matrices and the spectral decomposition is utilized in order to solve wave-propagation problems in a poroelastic *half-space*. In Section 2, the exact dynamic stiffness for the half-space and CFABCs are briefly reviewed. Representation of the half-space by CFABCs is proposed and its spectral characteristics are examined in Section 3. In Section 4, wave-propagation problems are solved using the developed numerical models. The paper will be summarized in Section 5.

2. Exact dynamic stiffness and continued-fraction absorbing boundary conditions for a poroelastic half-space

The governing equations for a poroelastic medium in a rectangular Cartesian coordinate system can be written using the generalized theory of Biot [1,2,32,33]:

For $i, j = x, y$, or z

$$\mu u_{i,jj} + (\lambda + \mu + Q\alpha^2)u_{j,ji} + Q\alpha w_{j,ji} - \rho \ddot{u}_i - \rho_w \ddot{w}_i = 0 \tag{1a}$$

$$Q\alpha u_{j,ji} + Q w_{j,ji} - \rho_w \ddot{u}_i - \frac{\rho_w}{n}\ddot{w}_i - f\dot{w}_i = 0 \tag{1b}$$

where u_i denotes the displacement of the solid skeleton; w_i the relative displacement of the fluid with respect to the solid skeleton multiplied by the porosity; λ and μ Lamé constants of the solid skeleton; Q and α parameters accounting for the compressibility of the two-phase medium; ρ_w the density of the fluid; $\rho = (1-n)\rho_s + n\rho_w$ the averaged density of the mixture in which ρ_s is the density of the solid; n the porosity; and $f = 1/\kappa$, in which κ is the permeability. Using the Helmholtz decomposition, solutions of the governing equations can be expressed in terms of potentials for P and S waves. Applying boundary and radiation conditions of a poroelastic half-space, its exact dynamic stiffness was derived [56,57]. Using the exact dynamic stiffness, wave-propagation problems in a poroelastic half-space can be solved by an integral transform technique [28,34]. However, since the exact dynamic stiffness is not a polynomial function of a horizontal wavenumber, it is difficult to utilize the spectral decomposition [45,46] in the solution of wave-propagation problems.

In order to simulate wave propagation in an infinite media, continued-fraction absorbing boundary conditions were developed [61,62]. Since the CFABCs can be cast as quadratic functions of a horizontal wavenumber, they can be effectively applied into the framework of the spectral decomposition [58]. Derivation of the CFABCs can be summarized as follows. A half-space is split into a layer and another half-space. Displacements are assumed to vary linearly in the layer and its dynamic stiffness is obtained following the Galerkin approach with the *mid-point* integration rule. Then, the layer-half-space system represents exactly the original half-space. Applying layers successively in this way, the half-space can be represented as a series of the layers. Since the dynamic stiffness of the layers can be expressed in a continued-fraction form, they can be referred to as continued fraction absorbing boundary conditions. Therefore, the exact dynamic stiffness of a half-space can be represented by the CFABCs.

Herein, the exact stiffness and CFABCs are briefly summarized for plane-strain and antiplane-shear problems, respectively.

2.1. Poroelastic half-space in plane strain

For a plane-strain condition, solutions of Eq. (1) are given in terms of potentials for $P1$, $P2$, and SV waves. Assuming x-harmonic and time-harmonic motions with wavenumber k and frequency ω, dynamic stiffness of the half-space in plane strain (Fig. 1a) can be obtained [56]:

$$\begin{Bmatrix} T_x(k,\omega) \\ iT_z(k,\omega) \\ iP(k,\omega) \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} U_x(k,\omega) \\ iU_z(k,\omega) \\ iW_z(k,\omega) \end{Bmatrix} \tag{2}$$