Comput. Methods Appl. Mech. Engrg. 263 (2013) 113–126

Contents lists available at SciVerse ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Locking-free isogeometric collocation methods for spatial Timoshenko rods

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ARTICLE INFO

Article history:
Received 7 November 2012
Accepted 20 March 2013
Available online 15 April 2013

Keywords:
Isogeometric analysis
Collocation methods
NURBS
Spatial Timoshenko rod
Locking-free methods

ABSTRACT

In this work we present the application of isogeometric collocation techniques to the solution of spatial Timoshenko rods. The strong form equations of the problem are presented in both displacement-based and mixed formulations and are discretized via NURBS-based isogeometric collocation. Several numerical experiments are reported to test the accuracy and efficiency of the considered methods, as well as their applicability to problems of practical interest. In particular, it is shown that mixed collocation schemes are locking-free independently of the choice of the polynomial degrees for the unknown fields. Such an important property is also analytically proven.

1. Introduction

Isogeometric Analysis (IGA), introduced by Hughes et al. [18,24], is a powerful analysis tool aiming at bridging the gap between Computational Mechanics and Computer Aided Design (CAD). In its original form IGA has been proposed as a Bubnov-Galerkin method where the geometry is represented by the spline functions typically used by CAD systems and, invoking the isoparametric concept, field variables are defined in terms of the same basis functions used for the geometrical description. This could be therefore viewed as an extension of standard isoparametric Finite Element Methods (FEM), where the computational domain exactly reproduces the CAD description of the physical domain. Moreover, recent works on IGA have shown that the high regularity properties of the employed functions lead in many cases to superior accuracy per degree of freedom with respect to standard FEM (cf., e.g., [12,19,25,31,32]). Given these unique premises, IGA has been adopted in different fields of Computational Mechanics, and the properties and advantages of this more than promising approach have been successfully tested and analyzed both from the practical and mathematical standpoints (see, among others, [2,5–11,13–15,20,22,26,28–30,33,34]).

The original basic concept of IGA (i.e., the use of basis functions typical of CAD systems within an isoparametric paradigm) can be also exploited beyond the framework of classical Galerkin methods. In particular, isogeometric collocation schemes have been recently proposed in [3], as an appealing high-order low-cost alternative to classical Galerkin approaches. Such techniques have also been successfully employed for the simulation of elastostatic and explicit elastodynamic problems [4] and their application to many other applications of engineering interest is currently the object of extensive research.

Within this context, a comprehensive study on the advantages of isogeometric collocation over Galerkin approaches is reported in [32], where the superior behavior in terms of accuracy-to-computational-time ratio guaranteed by collocation with respect to Galerkin is revealed. In the same paper, adaptive isogeometric collocation methods based on local hierarchical refinement of NURBS are introduced and analyzed, as well.

In view of the results briefly described above, isogeometric collocation clearly proposes itself as a viable and efficient implementation of the main IGA basic concepts.

In addition to this, isogeometric collocation has shown a remarkable and, to our knowledge, unique property when employed for the approximation of Timoshenko beam problems. In fact, it has been analytically proven and numerically tested in [16] that mixed collocation schemes for initially straight planar Timoshenko beams are locking-free without the need of any

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http://dx.doi.org/10.1016/j.cma.2013.03.009

compatibility condition between the selected discrete spaces. We highlight that this appealing property is deeply linked to the collocation approach adopted and not only a consequence of the isogeometric paradigm.

Moving along this promising research line, in the present paper, we aim at extending such results to the more interesting case of curved spatial Timoshenko rods of arbitrary initial geometry, limiting the discussion to the case of small displacements and displacement gradients.

The outline of the paper is as follows. In Section 2, we present the mechanical model of spatial Timoshenko rods and introduce the strong form formulation of the problem, in both displacement-based and mixed forms. Section 3 gives a brief review of one-dimensional B-Splines and NURBS as basis for isogeometric analysis. In Section 4, isogeometric collocation is introduced and collocation schemes for spatial Timoshenko rods are explained in detail. The proposed methods are tested on several numerical examples in Section 5, confirming their accuracy and showing possible applications. It is shown that collocation methods based on mixed formulations are locking-free for any choice of polynomial degrees for the different fields. This characteristic is also analytically proven by a theoretical convergence analysis in Section 6. Conclusions are finally drawn in Section 7.

2. The spatial Timoshenko rod equations

In the following we want to introduce a model describing a spatial rod, as, for example, the one reported in Fig. 1, which is clamped on the lower end and subjected to an external distributed load as well as concentrated loads and moments on the upper end. The model is developed under the assumptions of small displacements and displacement gradients, assuming a first-order Timoshenko-like shear deformation and following the approach proposed in [1].

2.1. Geometry description

The rod geometry is described by a spatial curve $\gamma(s)$. The curve parameter s is chosen to be the arc-length parameter and we denote with $()'$ the derivative with respect to the arc-length parameter, i.e., $()' = \mathrm{d}/\mathrm{d}s$. The unit tangent vector of the curve at a point $\gamma(s)$ is defined by

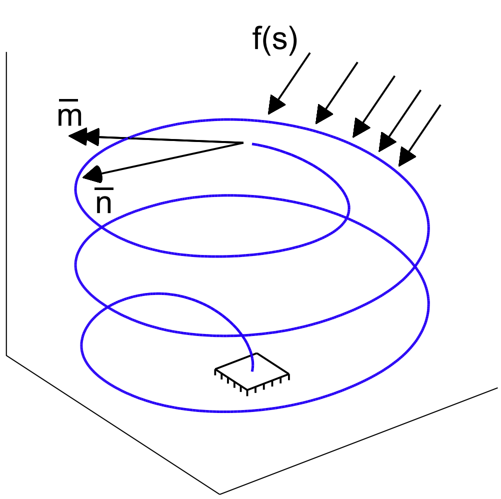

Fig. 1. Spatial rod model. The rod in this example is clamped on the lower end, subjected to a distributed external load $\mathbf{f(s)}$ and to concentrated external loads and moments, $\bar{\mathbf{n}}$ and $\bar{\mathbf{m}}$, respectively.

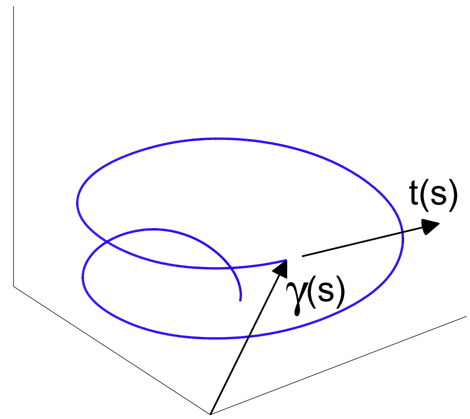

Fig. 2. Spatial curve description with $\gamma(s)$ as the position vector and $\mathbf{t}(s)$ as the tangent vector.

$$\mathbf{t}(s) = \gamma'(s) = \frac{\mathrm{d}\gamma(s)}{\mathrm{d}s}, \quad \text{for } s \in [0, l], \tag{1}$$

where $l > 0$ denotes the curve length. Fig. 2 shows a part of the curve of Fig. 1 from $\gamma(0)$ up to an arbitrary location $\gamma(s)$, along with the position vector and the unit tangent vector. In the following, all variables are assumed as functions of the arc-length parameter s (unless otherwise specified) also if we omit to explicitly indicate such a dependency.

2.2. Kinematic equations

Adopting a Timoshenko beam-like model, the rod deformation can be described by the centerline curve γ, a displacement vector $\mathbf{v}$, and a rotation vector $\boldsymbol{\varphi}$. Accordingly, we may introduce the generalized strains $\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$, respectively defined as the vector of translational (axial and shear) strains and the vector of rotational (torsional and flexural) strains. In particular, the translational strains are obtained by the first derivative of the displacements subtracting the rigid body rotations, whereas the rotational strains are obtained by the first derivatives of the rotations:

$$\boldsymbol{\varepsilon} = \mathbf{v}' - \boldsymbol{\varphi} \times \mathbf{t}, \tag{2}$$
$$\boldsymbol{\chi} = \boldsymbol{\varphi}'. \tag{3}$$

2.3. Constitutive equations

As usual for rod formulations, we introduce a vector $\mathbf{n}$ of internal forces and a vector $\mathbf{m}$ of internal moments. In the following, we assume a linear elastic constitutive relation in the form:

$$\mathbf{n} = \mathbb{C}\,\boldsymbol{\varepsilon}, \tag{4}$$
$$\mathbf{m} = \mathbb{D}\,\boldsymbol{\chi}. \tag{5}$$

Using an intrinsic basis, i.e., a basis composed by three orthogonal unit vectors with the first one equal to the tangent vector, the material matrices $\mathbb{C}$ and $\mathbb{D}$ are defined by:

$$\mathbb{C} = \text{diag}\,[EA, GA_1, GA_2], \tag{6}$$
$$\mathbb{D} = \text{diag}\,[GJ, EI_1, EI_2], \tag{7}$$

where E and G are the elastic and the shear modulus, A the cross sectional area, $A_1 = k_1A$ and $A_2 = k_2A$ (being k_1 and k_2 the shear correction factors), J the torsion constant and I_1 and I_2 the second moments of inertia. Within such a formulation, the components of $\mathbf{n}$ represent the axial force and the two components of the shear force, respectively, while the components of $\mathbf{m}$ represent the torsional moment and the two components of the bending moment, respectively.