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A high order homogenization model for transient dynamics of heterogeneous media including micro-inertia effects

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ABSTRACT

This manuscript presents a multi-dimensional, high order homogenization model for elastic composite materials subjected to dynamic loading conditions. The proposed model is derived based on the asymptotic homogenization method with multiple spatial scales. The high order homogenization model permits implementation using the standard finite elements and shape functions since it does not involve higher order terms typically present in dispersion models. The high order homogenization model can accurately capture wave dispersion in the presence of non-uniform density and non-uniform moduli within the material microstructure. Employing the Hybrid Laplace Transform/Finite Element Method, both displacement and traction boundary conditions for the macroscopic problem have been implemented. Finite element formulations for the first and second order influence functions, and the macroscale initial boundary value problem are presented. The performance of the model is verified by comparing model predictions to the local homogenization and the direct numerical simulations. The high order homogenization model is shown to predict the wave dispersion with very reasonable accuracy and cost. The proposed model can also capture the phononic bands – frequency bands within which the micro-inertia effects completely block wave propagation.

1. Introduction

This manuscript is concerned with computational modeling of wave propagation phenomena in composites and other heterogeneous materials. Composites are well known to exhibit favorable static properties such as high specific strength, specific stiffness, corrosion and fatigue resistance, and many others. These favorable properties are typically achieved by tailoring the constituent materials and the way they are put together; i.e., microstructural topology. Composites can also be tailored to control propagation of waves and dynamic properties [24]. Depending on the wave frequencies of interest, composites with superior functional and mechanical dynamic properties ranging from radar absorption [23] to impact and blast survivability [32] have been investigated. Such favorable dynamic properties in composites can be achieved by controlling wave dispersion generated by the local motion of the microstructure (i.e., micro-inertia).

When the length of the traveling waves and the size of the material microstructure are comparable, the waveform interacts with the microstructure through reflections and refractions (i.e., dispersion) at the interfaces of constituent materials with distinct material properties (e.g., moduli and density) [27,29]. The resulting overall dynamic response characteristics in the presence of dispersion is significantly different than those observed in an equivalent homogeneous medium

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characterized by homogenized moduli and density. The realization and first efforts on modeling of this phenomena dates back to the classical works of Cosserat and Cosserat [12], Mindlin [25], and Eringen and Suhubi [16] on nonlocal continuum theories.

Computational modeling of micro-inertia and dispersion is difficult because of the presence of multiple spatial scales. Three characteristic lengths are typically involved: the size of the microstructure (e.g., a unit cell or representative volume element), the length of the deformation and stress waves, and the size of the macroscopic structure or component of interest. The resolution of all microstructural features along the entire structure or component is clearly computationally prohibitive. One approach is to devise microstructure-based nonlocal effective medium theories. The effects of micro-inertia and dispersion have been recently modeled using gradient enhancement [7], time-harmonic Bloch expansions [30], scale bridging through Hamilton's principle [31], and models based on Mindlin's theory [15,19]. These approaches require the incorporation of high order strain and inertia gradient terms to the macroscopic equations of motion.

Computational homogenization based on the mathematical homogenization theory [4,8,21] is another alternative for modeling wave dispersions in heterogeneous materials. In order to capture the dispersion effects, it is necessary to include higher order terms in the asymptotic expansions. Chen and Fish [11] recognized the presence of numerical instability for large time windows and proposed a space–time homogenization model that regularizes the long-time behavior in the presence of dispersion. A stable homogenization model that does not require multiple time scales was devised in [17], where higher order equilibrium terms are included in the formulation. This rigorous homogenization model is valid for microstructures with constant mass density – the dispersions are generated due to contrast in moduli of the constituents only, and only in the presence of displacement boundary conditions. Hui and Oskay [22] derived a semi-analytical dispersive model for one-dimensional problems accounting for the wave dispersion in viscoelastic composite materials. Bakhvalov and Eglit [5] applied the mathematical homogenization theory to study wave propagation in thin heterogeneous plates. Andrianov et al. [1] provided analytical solutions by incorporating the high order homogenization modeling to investigate the wave dispersion in composite rods and square lattice of cylindrical inclusions. The two latter investigations focused on specific microstructural topologies. Recently, Fish et al. [18] proposed a new dispersion formulation, where the micro-inertia effects are introduced based on an eigenstrain formulation. This formulation was generalized to account for nonlinear behavior. Andrianov et al. [2,3] also addressed micro-inertia effects in nonlinear heterogeneous media using the homogenization method.

In this manuscript, we develop a new high order computational homogenization model that can capture the dispersion and micro-inertia effects in composites and other heterogeneous media subjected to dynamic loading conditions. The proposed model is based on the high order computational homogenization approach introduced in Ref. [17]. In particular, this manuscript provides the following novel contributions:

1. The proposed approach leads to a numerical model that can accurately capture wave dispersion in the presence of non-uniform density, in addition to non-uniform moduli, within the material microstructure. By this approach, the full range of impedance contrast can be interrogated.
2. The Hybrid Laplace Transform/Finite Element Method provides the capability to capture phononic band gaps by treating the response in the complex domain. This approach also enables the implementation of the traction boundary conditions, in addition to displacement boundary conditions.

Mathematical homogenization theory with multiple spatial scales is used to derive a higher-order homogenization model. The resulting model involves third- and fourth-order spatial derivatives and numerical implementation is not straightforward. An alternative simplified high order homogenization model without high-order spatial derivatives is introduced. The simplified high order homogenization model permits implementation using the standard finite elements and shape functions. The fourth order derivative terms are approximated using terms involving second order derivatives in time and space by exploiting asymptotic relationships, and by relating the high order moduli to the low order moduli through the use of Moore–Penrose pseudo-inverse. The performance of the high order homogenization model is assessed by comparing the model predictions to the direct numerical simulations. We note that the proposed homogenization-based model is valid when the characteristic length of the deformation and stress waves are larger (but not infinitely larger) than the size of the microstructure. When the size of the wavelengths of interest approaches to or smaller than the microstructural length scale, the wave interactions can only be captured by direct resolution of the microstructure.

Gradient elasticity models can also account for the effects of microinertia (e.g., [14,20]. A common way to address microinertia effects is by introducing sufficient number of length scales to the gradient elasticity model. If these parameters can be properly calibrated, the gradient elasticity models can also be powerful in mimicking the microinertia effects. However, in the multidimensional setting and particularly in case of complex microstructures, the authors are not aware of clear and systematic ways of identifying these length scales. In the proposed approach, the length scale is embedded in the acceleration modulus tensor, which is numerically computed as a function of the first and second order influence functions.

The remainder of this manuscript is organized as follows: Section 2 describes the dynamic problem of interest and the multiscale setting. Section 3 presents the formulation of the mathematical homogenization theory with multiple spatial scales and the resulting high order computational homogenization model. Section 4 provides the formulation of a simplified high order homogenization model that can be implemented using the standard finite element method. Section 5 presents the finite element formulation of the first and second order micro- and the macroscale problems for the implementation of the