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Robust topology optimization under loading uncertainty based on linear elastic theory and orthogonal diagonalization of symmetric matrices

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A R T I C L E I N F O

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A B S T R A C T

This paper proposes an efficient approach to solving robust topology optimization problem of structures under loading uncertainty. The objective is to minimize a weighted sum of the mean and standard deviation of structural compliance. Loading uncertainties can be in either concentrated loads or uniformly distributed loads. By exploiting of the linear elastic nature of structure, Monte Carlo sampling is completely separated from the topology optimization process, thus accurate calculation of objective function becomes possible. Efficient sensitivity analysis method is developed and its computational cost is only linearly proportional to the number of uncertain loads. The sensitivity analysis is also integrated into the density based topology optimization approach to solve the robust topology optimization problems. The numerical examples demonstrate the effectiveness of the proposed approach. The effect of uncertainty level, probability distribution of uncertainty and different influence of loading magnitude and directional uncertainty on the robust designs are also shown by the numerical examples.

1. Introduction

Structural topology optimization provides an efficient way for the development of new products to find effective design candidates at a very early stage of the design process [1]. Usually the design candidates are obtained under the assumed deterministic conditions, so they may become less efficient when considering various uncertainties in practical situations [2].

There exist various kinds of uncertainties in the design process, such as loading uncertainty [3–5], material uncertainty [6,7,2], geometric and boundary uncertainty [8–10]. Among all of them, loading uncertainty is often the most significant type for structural systems [3]. There are two standard approaches to considering loading uncertainty in structural topology optimization [11]. In the first approach, the optimal design should minimize the worst case objective with respect to uncertainty (worst-case approach) [12–16]; while in the second approach it should minimize the expected value of the objective function (average approach) [3–5,17,18]. We will focus on the average approach in this work.

The average approach can date back to the work of Ben-Tal and Bendsøe [19]. To design optimal truss structures that are robust under nodal load perturbation, a multiload model was introduced. In such model, several load scenarios were empirically picked and then a weighted average of the compliances associated with these loading scenarios is minimized. In essence, this model assigned a subjective discrete probability measure for each uncertain load scenarios. Evgrafov and

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Patriksson proposed a stochastic bilevel programming approach to solving truss topology optimization problems involving non-discrete probability measure [20], where the continuous probability measure is approximated by a sequence of discrete probability measures with finite support. By exploiting of the linear elastic nature of structures, Alvarez and Carrasco developed an explicit multiload-like formulation for truss topology optimization of minimum expected compliance design [17]. By this approach, all possible load scenarios are considered by their probability of occurrence, but only a few 'basis' load scenarios need to be solved, so it is both rational and computational efficient. The approximation approach and explicit multiload-like formulation were also proposed for continuum structural topology optimization problems considering loading uncertainty [18,3]. These explicit multiload-like formulations for truss and continuum structures represent the loading uncertainty by the resolution of the Cartesian components. Dunning et al. developed an analytical multiload-like approach [4], where the loading uncertainty is described by the probability distributions of the loading magnitude and direction. This analytical approach is computationally efficient, but it assumes all uncertainties are statistically independent and follow normal distributions. Zhao and Wang noticed the equivalence of the two representations of loading uncertainty and developed a more efficient multiload-like approach to solving the minimum expected compliance design problem, where the loading uncertainty can be described by any kind of practical probability distributions [21].

The average approach produces designs with good average performance. However, load scenarios with too large values of compliance may occur. To avoid this, stochastic model including variance of compliance was introduced [17,22]. The weighted sum of the expected and variance of compliance is taken as the objective function. A main difficulty in this formulation is how to efficiently compute the expected and variance of compliance and their derivatives. Carrasco et al. proposed explicit expressions for the expected and variance of compliance [5], where uncertainties are represented by the resolution of the Cartesian components and the components follow a multivariate normal distribution. These explicit expressions can be hardly generalized into problems where uncertainties are described by other kinds of distributions, since only for normal distributions, uncorrelation of random variables implies independence. Chen et al. employed the tensor product quadrature (TPQ) technique to estimate the expected and variance of the compliance [7], where the uncertainties are represented by loading magnitude and direction. The TPQ approximation method is much more efficient than the Monte Carlo method. However, the well known 'curse of dimensionality' makes it impractical [23]. Recently, Kim and Guyer developed an analytical formulation for mean and variance of compliance considering generalized uncertainty in both loading magnitude and direction [24]. Although it seems efficient, the formulation is long and tedious. In addition, generalizing it into 3D problems will be very complicated. Zhao and Wang suggested an efficient approach to calculating the mean and variance of compliance considering any practical loading uncertainty [21]; however, the computational cost of sensitivity analysis is still heavy, especially when the number of uncertain loads is large.

In this paper, a density based approach for robust topology optimization problem of structures under loading uncertainty is presented. The objective is to minimize a weighted sum of the mean and standard deviation of compliance. The loading uncertainty can be in either concentrated loads or uniformly distributed load. The loading uncertainty can be described by any practical joint probabilistic distributions. By exploiting of the linear elastic nature of structure, only a small subset of load scenarios need to be considered at each iteration of the optimization process. Massive Monte Carlo sampling is completely separated from the iteration process of topology optimization and efficient calculation of objective function becomes possible. Sensitivity analysis method is also developed by using orthogonal diagonalization of symmetric matrices. The computational cost of the sensitivity analysis method is only linearly proportional to the number of uncertain loads, so it is quite efficient. The sensitivity analysis is also integrated into the density based topology optimization approach to solve the robust topology optimization problem of structures under loading uncertainty. Our discussion will focus on 2D topology optimization problems, but can be readily generalized to 3D cases.

This paper is organized as follows: Section 2 presents the problem formulation and density based approach to robust topology optimization of structures under loading uncertainty; Section 3 develops efficient solution method, including calculation of objective function, sensitivity analysis, filtering method, optimization algorithm and numerical implementation and Section 4 gives some numerical examples to demonstrate the proposed approach. Concluding remarks are given in the last section.

2. Robust topology optimization of structures under loading uncertainty

2.1. Robust topology optimization problem formulation

The formulation of the robust topology optimization problem considering loading uncertainty is given as (1). The objective is to minimize a weighted sum of the mean and standard deviation of compliance. Here only surface loading is considered. ω is a realization in the sample space Θ and it is used to indicate that the involved physical quality is random.

$$\begin{aligned} \text{Min}: \quad & J = \mu(c) + \beta\sigma(c) \\ \text{s.t.}: \quad & \int_D A\varepsilon(u(\omega))\varepsilon(v(\omega))d\Omega = \int_\Gamma f(\omega)v(\omega)d\Gamma, \\ & \int_D d\Omega \leqslant V_{\max}, \end{aligned} \tag{1}$$