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A Bayesian approach to selecting hyperelastic constitutive models of soft tissue

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Abstract

Hyperelastic constitutive models of soft tissue mechanical behavior are extensively used in applications like computer-aided surgery, injury modeling, etc. While numerous constitutive models have been proposed in the literature, an objective method is needed to select a parsimonious model that represents the experimental data well and has good predictive capability. This is an important problem given the large variability in the data inherent to soft tissue mechanical testing.

In this work, we discuss a Bayesian approach to this problem based on Bayes factors. We propose a holistic framework for model selection, wherein we consider four different factors to reliably choose a parsimonious model from the candidate set of models. These are the qualitative fit of the model to the experimental data, evidence values, maximum likelihood values, and the landscape of the likelihood function. We consider three hyperelastic constitutive models that are widely used in soft tissue mechanics: Mooney–Rivlin, Ogden and exponential. Three sets of mechanical testing data from the literature for agarose hydrogel, bovine liver tissue, porcine brain tissue are used to calculate the model selection statistics. A nested sampling approach is used to evaluate the evidence integrals. In our results, we highlight the robustness of the proposed Bayesian approach to model selection compared to the likelihood ratio, and discuss the use of the four factors to draw a complete picture of the model selection problem.

Keywords: Bayesian; Uncertainty quantification; Soft tissue constitutive model; Hyperelastic; Nested sampling; Model selection

1. Introduction

1.1. Motivation

Hyperelastic constitutive models of soft tissue mechanical behavior find widespread use in applications like traumatic brain injury simulation, computer-aided surgery and functional tissue engineering. These models are generally phenomenological in nature and are derived from a strain energy density function that is postulated to depend on invariants of the deformation field in the elastic body. The model parameters are usually determined by a

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least squares fit of the constitutive model to mechanical testing data from experiments such as compression, tension [1–3], indentation [4,5], aspiration [6,7], etc.

For most problems, the choice of the constitutive model is driven by the quality of the fit (e.g., R^2 value) and the analyst's experience. Commonly used models include those based on strain invariants (e.g., Neo-Hookean, Mooney–Rivlin, exponential, Pucci–Saccomandi, Arruda–Boyce, etc.), or on the principal stretches (e.g., Ogden, Peng–Landel, etc.). See [8,9] for a comprehensive list. Given this wide range of available models, an objective approach is needed to ensure that the chosen constitutive model strikes a balance between fit quality, predictive capability, and complexity. A simple way to measure the model's complexity is through the dimensionality of its parameter space. A model that is too simple and has too few parameters may not faithfully represent the experimental mechanical response, while a model that is overly complex may be of limited use or have so many parameters that they cannot be easily identified from the available data. Thus the issue of parameter estimation is an integral part of the model selection process since the model is complete only after the parameters are estimated.

Another issue of concern in soft tissue mechanics is that the experimental data almost always have high variability, which leads to large uncertainties in the constitutive model parameters. This large variability is due to many factors: (i) soft tissue is a complex material that can be highly heterogeneous; (ii) patient to patient differences can be large; (iii) soft tissue is difficult to procure, handle and preserve, and obtaining good specimens is challenging; and (iv) experimental protocols vary widely between laboratories. These difficulties are exacerbated when the problem of interest involves irreversible and discontinuous phenomena like tissue fracture and damage, a common occurrence in injury biomechanics [10]. This variability cannot be ignored and should be systematically included in the parameter estimation and model selection process in order to obtain a reliable constitutive model.

1.2. Model selection strategies

Model selection procedures often rank the candidate models based on some widely accepted criteria. The selected model should obey the principle of parsimony or Occam's razor, which states that the simplest model that can explain the data should be accepted. That is, a balance between goodness of fit and model complexity has to be achieved. This property of the model is essential to avoid overfitting and to render the model more testable. Most model selection methods can be categorized into three groups: (a) Bayesian methods using the Bayes factor [11,12], (b) frequentist methods that include p-value method, Mallows's C_p, etc., and (c) Information-theoretic methods like the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and variants [13].

The Bayesian approach has some advantages over frequentist methods [14]. First, it is easier to interpret the posterior probabilities of the model and the Bayes factor as the odds of one model over the other. Second, the Bayesian approach is consistent in the sense that it guarantees the selection of the true model if it is part of the candidate model set under very mild conditions. If the true model is not present in the set, then the model closest to the true model in terms of the Kullback–Leibler divergence is chosen [15,16].

The Bayesian approach mainly involves updating prior knowledge of the model with new experimental observations (represented by a likelihood function) to obtain the posterior distribution of the model parameters. The marginal likelihood or evidence is then obtained by marginalizing the likelihood function over the space of its existence; this is discussed in more detail in Section 3.

Bayesian model selection is a natural Occam's razor. The use of the marginal likelihood automatically penalizes overly complex models because complex models spread their probability mass very widely and predict that everything is possible. Thus the probability of the actual data is small. This is illustrated in Fig. 1 (adapted from [17]), where the model space $\mathfrak{M}$ consists of three models $\boldsymbol{m}_1, \boldsymbol{m}_2, \boldsymbol{m}_3$ with equal probabilities, each with a single parameter θ. The data space $\mathfrak{D}$ is assumed to be one-dimensional, and $\boldsymbol{d} \in \mathfrak{D}$ is the dataset used to estimate the parameter θ, which is assumed to be a single value. Model $\boldsymbol{m}_3$ is a more complex model in the sense that it can predict more data, while model $\boldsymbol{m}_1$ is the simplest. The dashed straight line corresponds to a particular observed dataset $\boldsymbol{d}$. The quantity $P(\boldsymbol{d}|\boldsymbol{m}_i)$ represents the probability of obtaining the data given a particular model $\boldsymbol{m}_i$; this is the marginal likelihood or evidence. The quantity $P(\theta|\boldsymbol{d}, \boldsymbol{m}_i)$ represents the posterior probability of the parameter given the dataset and model. For the example shown here, model $\boldsymbol{m}_1$ is unable to predict the data, model $\boldsymbol{m}_2$ predicts the data and has the highest evidence value, while model $\boldsymbol{m}_3$ also predicts the data but with a lower evidence value. Thus model $\boldsymbol{m}_2$ is the model of choice.