Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 291 (2015) 123–145
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Fast Bayesian optimal experimental design for seismic source inversion

Quan Long[a,c,*], Mohammad Motamed[b], Raúl Tempone[a]

[a] *SRI Center for Uncertainty Quantification in Computational Science and Engineering, King Abdullah University of Science and Technology, Thuwal, Saudi Arabia*
[b] *Department of Mathematics and Statistics, The University of New Mexico, USA*
[c] *Institute for Computational Engineering and Sciences, The University of Texas at Austin, USA*

Received 30 July 2014; received in revised form 2 February 2015; accepted 25 March 2015
Available online 1 April 2015

Abstract

We develop a fast method for optimally designing experiments in the context of statistical seismic source inversion. In particular, we efficiently compute the optimal number and locations of the receivers or seismographs. The seismic source is modeled by a point moment tensor multiplied by a time-dependent function. The parameters include the source location, moment tensor components, and start time and frequency in the time function. The forward problem is modeled by elastodynamic wave equations. We show that the Hessian of the cost functional, which is usually defined as the square of the weighted L_2 norm of the difference between the experimental data and the simulated data, is proportional to the measurement time and the number of receivers. Consequently, the posterior distribution of the parameters, in a Bayesian setting, concentrates around the "true" parameters, and we can employ Laplace approximation and speed up the estimation of the expected Kullback–Leibler divergence (expected information gain), the optimality criterion in the experimental design procedure. Since the source parameters span several magnitudes, we use a scaling matrix for efficient control of the condition number of the original Hessian matrix. We use a second-order accurate finite difference method to compute the Hessian matrix and either sparse quadrature or Monte Carlo sampling to carry out numerical integration. We demonstrate the efficiency, accuracy, and applicability of our method on a two-dimensional seismic source inversion problem.

Keywords: Bayesian experimental design; Information gain; Laplace approximation; Monte Carlo sampling; Seismic source inversion; Sparse quadrature

1. Introduction

In seismic source inversion, the source parameters can be estimated based on minimizing a cost functional, which is usually given by the weighted L_2 norm of the difference between the recorded and simulated data. The simulated

* Corresponding author at: SRI Center for Uncertainty Quantification in Computational Science and Engineering, King Abdullah University of Science and Technology, Thuwal, Saudi Arabia.
E-mail addresses: quan.long@kaust.edu.sa, quan@ices.utexas.edu (Q. Long), motamed@math.unm.edu (M. Motamed), raul.tempone@kaust.edu.sa (R. Tempone).

http://dx.doi.org/10.1016/j.cma.2015.03.021

data are obtained by solving a complex forward model, which is described by a set of elastic wave equations. The recorded data are usually the time series of ground displacements, velocities and accelerations, recorded by an array of receivers (seismographs) on the surface of the ground and in observation wells. On the other hand, if we treat the source parameters as random variables, we seek a complete statistical description of all parameter values that are consistent with the noisy measured data. This can be achieved using a Bayesian approach [1] by formulating the inverse problem as a statistical inference problem, incorporating uncertainties in the measurements, the forward model, and any prior information about the parameters. The solution of this inverse problem is the set of posterior probability densities of the parameters updated from prior probability densities using Bayes theorem. Meanwhile, the maximum a posteriori (MAP) estimation is obtained by minimizing a cost functional, defined as the negative logarithm of the posterior.

Considering the financial and logistic costs of collecting real data, it is important to design an optimal data acquisition procedure, with the optimal number and locations of receivers. In the current work, we assume that there is additive Gaussian measurement noise and model the seismic source by a point moment tensor multiplied by a time-dependent function. The parameters include the source location, moment tensor components, and start time and frequency in the time function. There are in total $N_\theta = 7$ parameters in the two-dimensional model and $N_\theta = 11$ parameters in a three-dimensional model. We then consider the problem of optimal experimental design in a Bayesian framework. Under this Bayesian setting, a prior probability density function (pdf) of the source parameters is given based on expert opinion and/or historical data, and the effect of the measured data is incorporated in a likelihood function. A posterior pdf of the parameters is then obtained through Bayes theorem by the scaled product of the prior pdf and the likelihood function. To measure the amount of information obtained from a proposed experiment, we use the expected Kullback–Leibler divergence, also called the expected information gain. It is specifically defined as the marginalization of the logarithmic ratio between the posterior pdf and prior pdf over all possible values of seismic source parameters and the data. The optimal experimental setup will then be the one that maximizes the expected information gain. Finding such an optimal experiment requires calculating the expected information gains corresponding to many possible setups. See [2] for more details.

The common method for estimating the expected information gain is based on sample averages, which leads to a double-loop integral estimator [3,4]. This approach can be prohibitively expensive when the simulated data are related to solutions of complex partial differential equations (PDEs). Hence, in such cases, such as seismic source inversion, more efficient approaches are required.

In this paper, we develop a new technique for efficiently computing the expected information gain of non-repeatable experiments arising from seismic source inversion. The efficiency of the new approach, which is based on our recent work in [5], results from the reduction of the double-loop integration to a single-loop one. This reduction can be accurately performed by Laplace approximation when the posterior distribution of the source parameters is concentrated. As the main contribution of the current work, we show that the posterior pdf concentrates around “true” source parameters, due to the fact that the Hessian of the cost functional is proportional to the number of receivers and measurement time. Consequently, the error of our approximation diminishes by increasing the number of receivers and recording time and by improving the precision of our measurements. Hence, we extend the methodology in [5] from repeatable static problems to non-repeatable time-dependent problems, e.g., earthquakes. From a mathematical point of view, we seek the concentration of posterior probability distribution conditioned on a time series of data instead of repetitive experiments. We also carry out a rescaling of the original parameters to address the issue of an ill-conditioned Hessian matrix stemming from the large span of the parametric magnitudes in the seismic source term. The integrand of the approximated expected information gain is a function of the rescaled posterior covariance matrix, which can be obtained by solving $N_\theta + 2$ forward problems, consisting of $N_\theta + 1$ primal problems and 1 dual problem.

The remainder of this paper is organized in the following way. In Section 2, we formulate the experimental design problem for seismic source inversion and briefly introduce the cost functional and the expected information gain for a given experimental setup in the Bayesian setting. We present the approximated form of the expected information gain based on Laplace approximation and derive the rate of errors in Section 3. In the same section, we also summarize the finite difference method for solving the forward problems, the adjoint approach for obtaining the Hessian matrix, and the sparse quadratures and Monte Carlo sampling for numerical integration. In Section 4, we consider numerical examples for a simplified earthquake and design optimal experiments. Conclusions are presented in Section 5.