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Verification and validation of a Direct Numerical Simulation code

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Highlights

- Verification of a boundary layer numerical code by the Method of Manufactured Solutions.
- Convergence order analysis of a boundary layer numerical code.
- Validation of a boundary layer numerical code through comparison with experimental data.
- Agreement between boundary layer numerical code and Linear Stability Theory results.
- Use of high-order approximations.

Abstract

The verification of a Direct Numerical Simulation code is carried out using the Method of Manufactured Solutions. Numerical results from the code are also compared with experimental and Linear Stability Theory results in a boundary layer over an airfoil. Displacement thickness, momentum thickness and shape factor are used to measure the boundary layer. Comparisons considering the amplitude of the velocity disturbance caused by two-dimensional Tollmien–Schlichting waves are also made. The results show the verification and validation of the Direct Numerical Simulation code.

MSC: 00-01; 99-00

Keywords: Boundary layer flow; High-order compact finite-difference scheme; Direct Numerical Simulation; Verification and validation; Method of Manufactured Solutions

1. Introduction

Numerical studies involving fluid motion have large applications in science and engineering. As an example, computational simulations can be used for climate predictions, aerodynamics and oil industry, biomedical engineering, and many other areas. Due to the great importance of these numerical predictions for practical applications, the

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credibility of mathematical models and numerical methods is a factor that should be investigated. Verification and validation is the field of study that provides different techniques to quantify how reliable a simulation code is [1–3].

Particularly, the simulation of boundary layer flows has been the focus of many researchers in Computational Fluid Dynamics (CFD). The no-slip condition on a solid body reduces the velocities of the outer flow to zero on the surface. This reduction generates large velocity gradients in a thin layer adjacent to the body surface. This thin region is the boundary layer, in which strong viscous effects exist [4]. The boundary layer is directly affected by the outer flow [5,6]. For example, the presence of a favorable pressure gradient causes the outer flow to accelerate and stabilizes the boundary layer flow. On the other hand, the presence of an adverse pressure gradient decelerates the outer flow and increases the instability of the boundary layer. An increasing instability leads to transition to turbulence. Experimental and numerical studies are being used to explain and predict this phenomenon [7–10].

In this paper, we adopt a Direct Numerical Simulation (DNS) code to predict laminar–turbulent transition problems in incompressible boundary layer flows. The results carried out by this code are verified through the Method of Manufactured Solutions (MMS). This is the most efficient method for verification of codes. The basic idea is producing a solution and transforming the original set of governing equations into a set of similar equations where the exact solution is available.

A comparison of the amplification rate of Tollmien–Schlichting waves in a boundary layer to Linear Stability Theory (LST) and to experimental results were also used to verify and validate the code. Unsteady disturbances of small amplitude produce Tollmien–Schlichting (TS) waves in the flow. The growth or decay of these waves evidences the flow stability. According to LST, when the amplitudes of these waves are infinitesimal, they can grow until certain point (neutral curve) and start to decay. Based on this, TS waves were generated in the flow in order to study their effect in the velocity profile.

The governing equations adopted here are the incompressible Navier–Stokes equations written in vorticity–velocity formulation. The numerical method is based on high-order finite-difference approximations for the discretization of the streamwise and wall-normal spatial derivatives. In the spanwise direction a spectral method that uses Fast Fourier Transformation is applied. The time integration is carried out by a fourth-order Runge–Kutta scheme.

2. Formulation

In this section the governing equations, the reference system, and the integration domain are presented.

2.1. Governing equations

The governing equations are the dimensionless Navier–Stokes equations for incompressible flow with constant viscosity written in orthogonal coordinates. The non-dimensionalization is made considering the Reynolds number *Re*, given by:

$$Re = \frac{\tilde{U}_\infty \tilde{L}}{\tilde{\nu}},$$

where $\tilde{U}_\infty$ is the reference velocity, $\tilde{L}$ is the reference length and $\tilde{\nu}$ is the kinematic viscosity. The reference velocity is the free stream velocity and the reference length is the distance from the leading edge, the foremost edge of a rigid body (e.g. airfoil and flat plate).

The dimensionless variables are written as:

$$x = \frac{\tilde{x}}{\tilde{L}}; \quad y = \frac{\tilde{y}}{\tilde{L}}; \quad z = \frac{\tilde{z}}{\tilde{L}}; \quad u = \frac{\tilde{u}}{\tilde{U}_\infty}; \quad v = \frac{\tilde{v}}{\tilde{U}_\infty}; \quad w = \frac{\tilde{w}}{\tilde{U}_\infty};$$
$$\omega_x = \frac{\tilde{w}_x \tilde{L}}{\tilde{U}_\infty}; \quad \omega_y = \frac{\tilde{w}_y \tilde{L}}{\tilde{U}_\infty}; \quad \omega_z = \frac{\tilde{w}_z \tilde{L}}{\tilde{U}_\infty}; \quad t = \frac{\tilde{t}\tilde{U}_\infty}{\tilde{L}},$$

where the tilde denotes dimensional variables; x, y and z are the spatial coordinates in streamwise, wall-normal and spanwise directions, respectively; u, v, w and ω_x, ω_y, ω_z are the velocity and vorticity components in each direction, respectively; and t is the time.