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Isogeometric Kirchhoff–Love shell formulations for general hyperelastic materials

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Abstract

We present formulations for compressible and incompressible hyperelastic thin shells which can use general 3D constitutive models. The necessary plane stress condition is enforced analytically for incompressible materials and iteratively for compressible materials. The thickness stretch is statically condensed and the shell kinematics are completely described by the first and second fundamental forms of the midsurface. We use C^1-continuous isogeometric discretizations to build the numerical models. Numerical tests, including structural dynamics simulations of a bioprosthetic heart valve, show the good performance and applicability of the presented methods.
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1. Introduction

Thin shells can undergo large displacements and rotations while exhibiting only small strains, especially for bending-dominated deformations, due to their geometric dimensions. Accordingly, a geometrically nonlinear approach is often employed, where nonlinear kinematics are accounted for but a linear strain–stress relation is assumed, corresponding to the St. Venant–Kirchhoff constitutive model. However, this approach is not appropriate in the presence of large membrane strains and when nonlinear elastic constitutive laws, typically used for the modeling of rubber-like materials and biological tissues, need to be employed. In such cases, a fully nonlinear formulation, including both kinematic and constitutive nonlinearities, needs to be adopted.

It is well known that thin shells can be modeled appropriately with the classical Kirchhoff–Love kinematics, but the necessary C^1 continuity inherent in such models has always been a major obstacle for the development of efficient

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finite element formulations. As a consequence, thick shell formulations based on Reissner–Mindlin kinematics requiring only C^0 continuity are much more widespread in finite element shell analysis [1]. In the context of finite strains, higher order shell models including transverse normal strains [2–6] or solid-shells [7–9], just to name a few, are usually employed since they facilitate the implementation of general 3D material laws. As a matter of fact, the formulation of C^1 conforming thin shell finite elements is possible and has been presented, e.g., in [10,11], including also finite strains. However, these elements are very complicated and computationally expensive (in the mentioned references, triangles with 54 degrees of freedom per element have been used) and, therefore, of little practical use. A possible way to use C^0 elements in thin shell formulations is to compute curvatures in an approximative way by the surface normals of surrounding elements, see [12,13]. Alternative, smooth discretization techniques like meshless methods and subdivision surfaces allow a very natural implementation of thin shell models, see [14] for a meshless implementation and [15,16] for the subdivision surfaces approach.

Isogeometric analysis (IGA) [17] is a new trend in computational mechanics, which can be considered as an extension of finite element analysis where functions typically used in Computer Aided Design (CAD) are adopted as basis functions for analysis. The most widespread functions in both CAD and IGA up to today are Non-Uniform Rational B-Splines (NURBS). An interesting alternative is T-splines [18,19], which allow for local refinement and watertight modeling and have also been applied successfully in the context of IGA, see e.g. [20–23]. While the initial motivation of IGA was to better integrate design and analysis by this common geometry description, it has also been found in various studies that IGA has superior convergence properties compared to classical finite elements on a per degree-of-freedom basis [24–26]. Over the last years, IGA has attracted enormous interest in nearly all fields of computational mechanics and it also gave new life to the development of shell formulations, including rotation-free shells [27–29], Reissner–Mindlin shells [30–33], blended shells [34], hierarchic shells [35], and solid shells [36–40]. The high continuity naturally inherent in the isogeometric basis functions allows for a straightforward implementation of C^1 thin shell models. In [27], an isogeometric formulation for geometrically nonlinear Kirchhoff–Love shells has been firstly presented. The formulation is rotation-free and purely surface-based, which means that the shell kinematics are completely described by the midsurface's metric and curvature properties. This also allows for a direct integration of IGA into CAD systems, which are usually based on surface geometry models [41,42]. The lack of rotational degrees of freedom also permits a direct coupling of structures and fluids in fluid–structure interaction (FSI) applications, see [43,22,44]. Furthermore, this shell model has been applied to wind turbine blade modeling [45,46], isogeometric cloth modeling [47], explicit finite strain analysis of membranes [48], and for the modeling of fracture within an extended IGA approach [49].

In the present paper, we extend the isogeometric shell model presented in [27] to the large strain regime, including compressible and incompressible nonlinear hyperelastic materials. We develop the formulations such that arbitrary 3D constitutive laws can be used for the shell analysis. The transverse normal strain, which cannot be neglected in the case of large strains, is statically condensed using the plane stress condition (in this paper we adopt the commonly accepted, although incorrect, use of the term "plane stress" for referring to the state of zero transverse normal stress). As a consequence, the thickness stretch is not considered as additional variable and the shell kinematics are still completely described by the metric and curvature variables of the midsurface. The imposition of the plane stress condition is done differently for compressible and incompressible materials. While for the former it is obtained by an iterative update of the deformation tensor, it can be solved analytically for the latter by using the incompressibility constraint. In both approaches we derive the formulations considering a general 3D strain energy function, such that arbitrary 3D constitutive models, both compressible and incompressible, can be used for the shell formulation straight away. We present the derivation from the continuum to the shell model in detail using index notation in a convective curvilinear frame.

The paper is structured as follows: In Section 2, we introduce some notation convention used in this paper. Section 3 presents geometrical basics for the shell description while in Section 4, the shell kinematics are derived. In Section 5, the constitutive equations are presented with a focus on the consistent derivation from the 3D continuum to the shell model via the plane stress condition. In Section 6, we show the variational formulation, with detailed linearization of the strain variables to be found in Appendix C. In Section 7, we discuss the isogeometric discretization and implementation details. In Section 8, we present numerical tests including benchmark examples for which analytical solutions are available, as well as the application to biomechanics problems, namely structural dynamics simulations of a bioprosthetic aortic valve, which demonstrate the validity and applicability of the presented methods. Finally, conclusions are drawn in Section 9.

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